

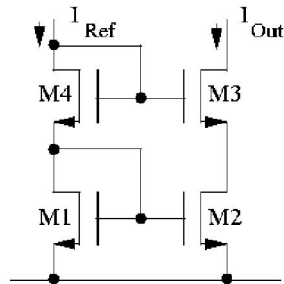
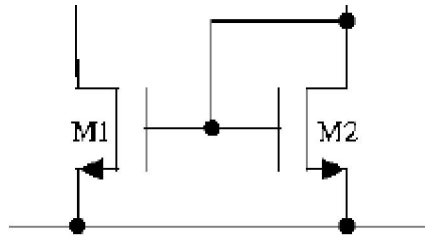
Welcome to  
0510.7720.01 Winter semester 2021  
Mixed Signal Electronic Circuits  
Instructor: Dr. M. Moyal

**Lecture 6 and 7- recorded for: may 13<sup>th</sup> and 20  
2021..**

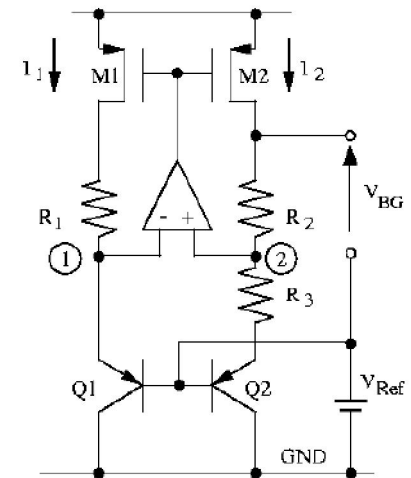
**Comparator: Operation and Design**

# Quick review on analog circuits basics

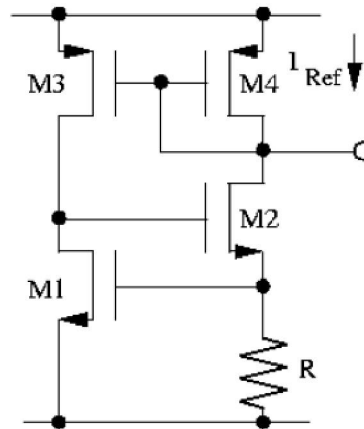
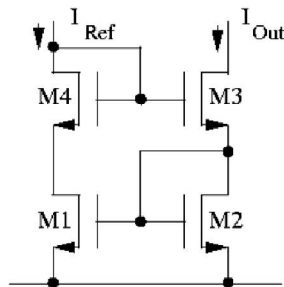
- Comparator Basics
- Architectures
- Error Sources
- Comparator Examples



**Cascode/cascade**

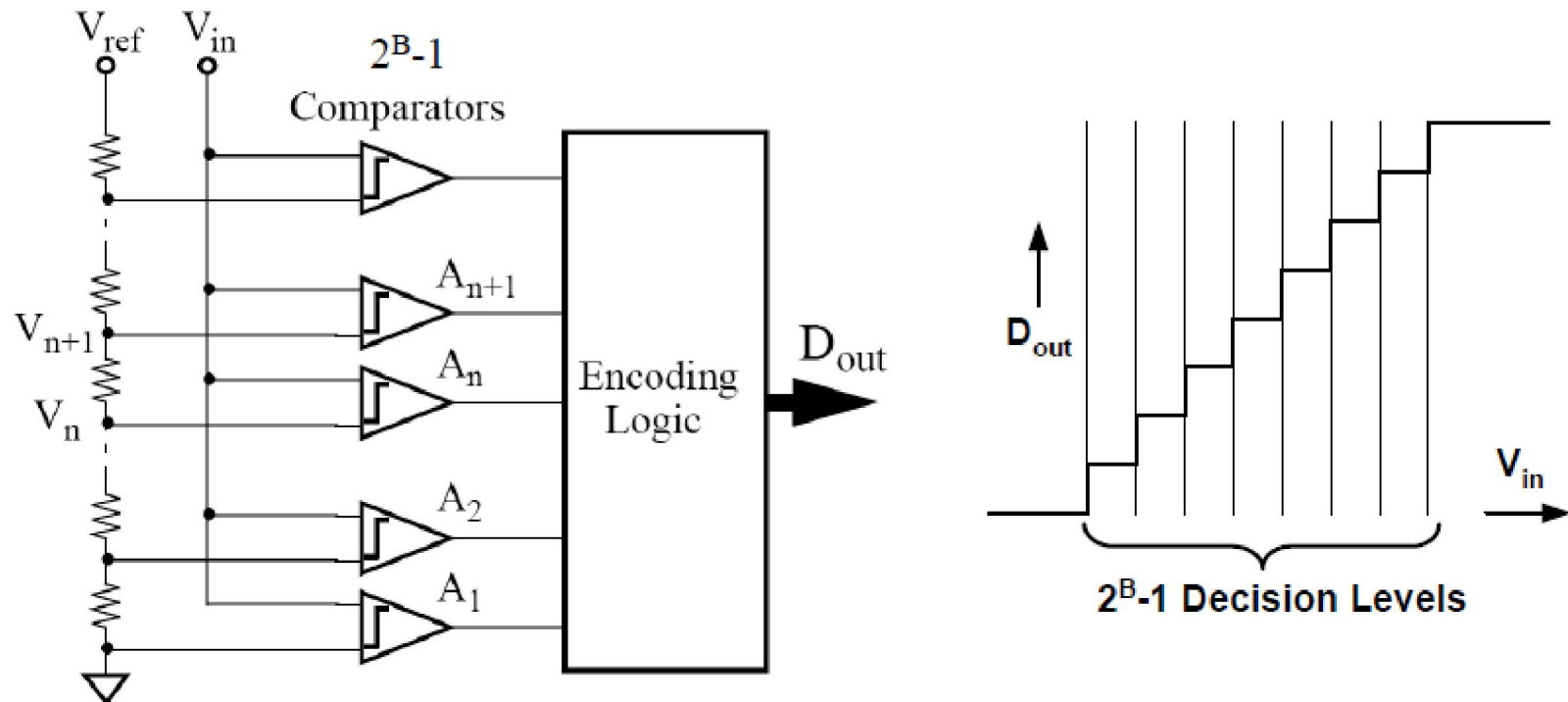


**Improved Wilson Current Mirror**



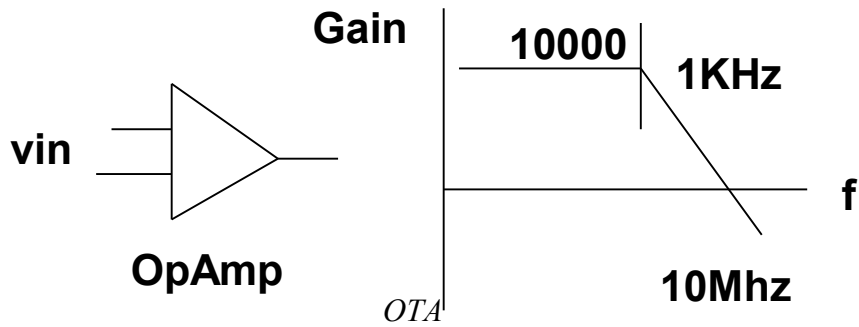
For the project assignment---convert the digital back to numbers..

# Flash ADC



- Comparator:
  - Non Sampled
  - Sampled
  
- Error in Comparators
  
- Basic Analysis
  
- Architectures
  
- Special topics – Calibrated and differential

# Speed and Feedback Basics

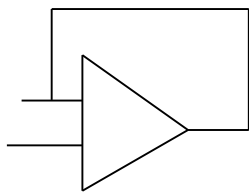


Speed

Gain(dc)

$$\tau = 1ms$$

80dB

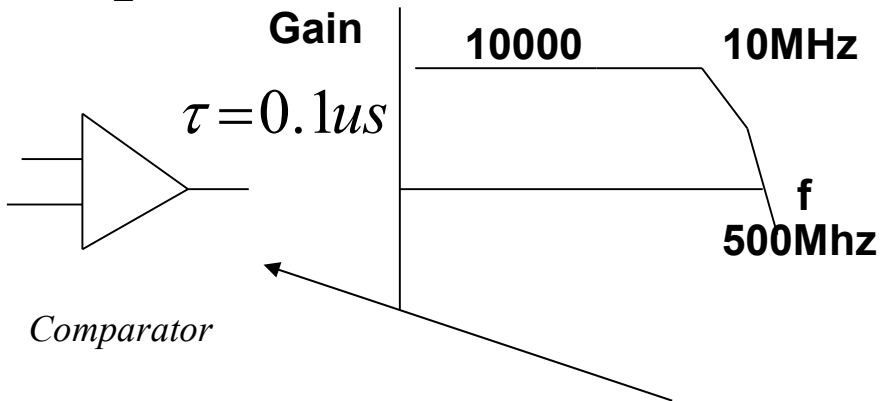


Gain=1  
And accurate

$$\tau = 0.1\mu s$$

0dB

Feed \_ back



$$\tau \sim 0.1\mu s$$

80dB

Removed all compensation

### ❑ **Comparator:**

- ❑ A Link from Analog to Digital – Quantizer

### ❑ **Definition:**

- ❑ Compares between 2 or more inputs and produce a digital value high or low- ***“Its a 1 bit ADC !”***

### ❑ **Structure:**

- ❑ A chain of gain stages (no feed back-unlike amplifier)

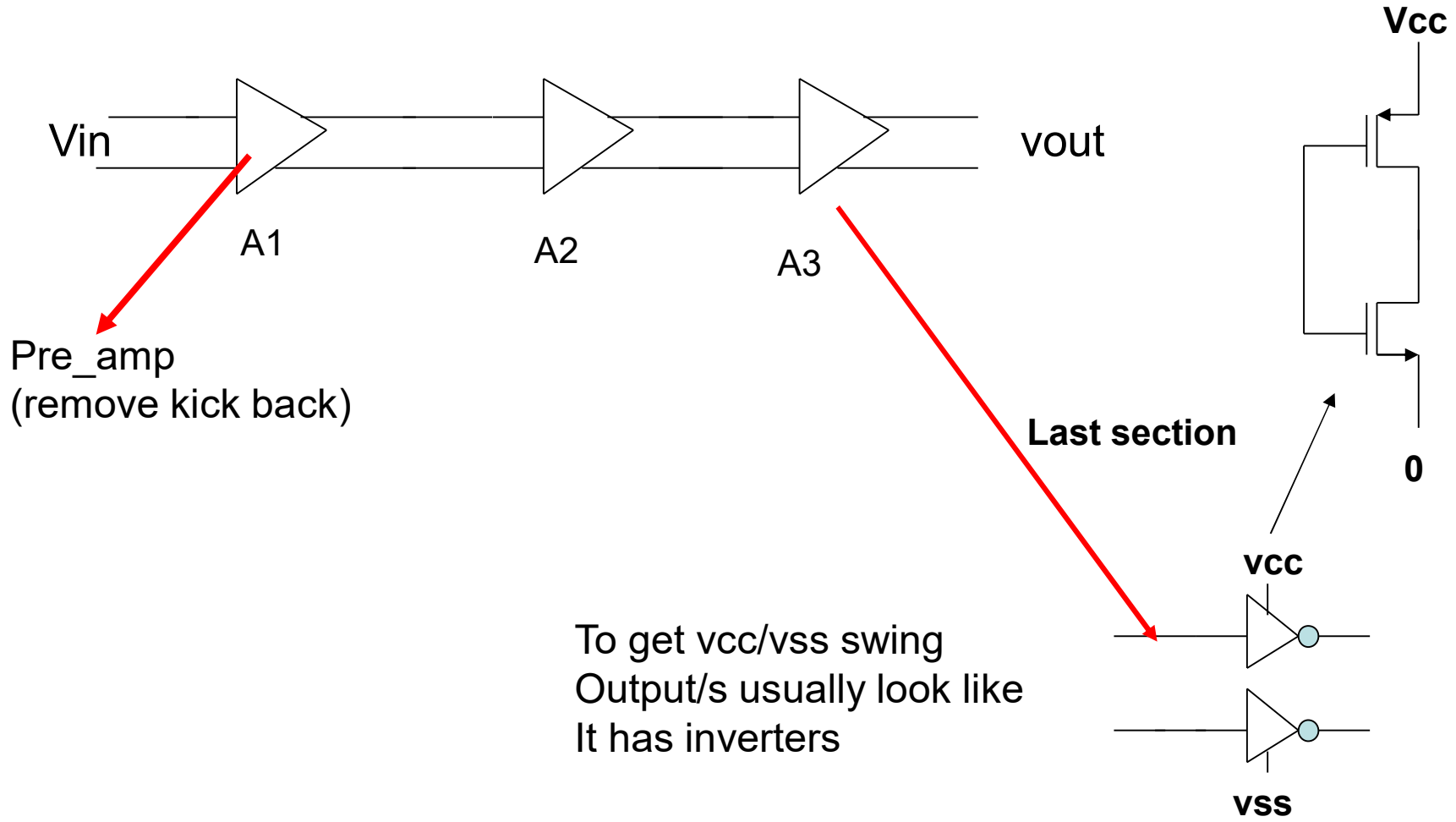
- ❑ It is used in an “open Loop” configuration to achieve fast digital response (opamps are slow and big)

# Basic Architecture – 1 of Comparators



## ❑ Non Sampled: Continuous Time (CT)

- ❑ Output is gain time input differences:
  - ❑ slow because internal nodes need to be recovered



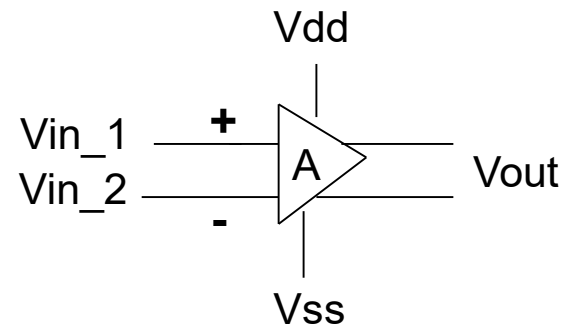


## Mathematical Descriptions – Low Frequency

For small  $V_{in1} - V_{in2}$

$$V_{out} = A(V_{in_1} - V_{in_2})$$

A = Gain of the combine structure



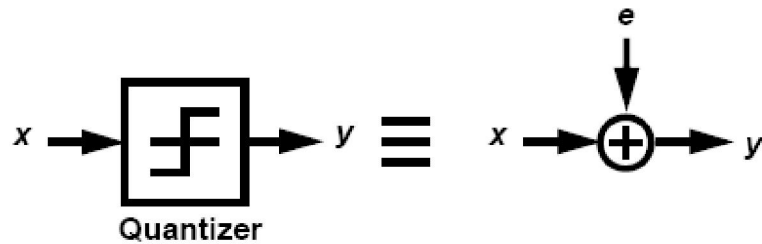
For all  $V_{in1} - V_{in2}$        $V_{out}$  = clamp at either supply

### ***Ideal***

$V_{out} = A(V_{in_1} - V_{in_2}) \rightarrow$  results fall between supplies (realistically its exponential)

$$V_{out} = V_{dd} \quad \text{if } A(V_{in_1} - V_{in_2}) > V_{dd} - V_{ss}$$

$$V_{out} = V_{ss} \quad \text{if } A(V_{in_1} - V_{in_2}) < V_{dd} - V_{ss}$$



exact model if  $e$  is defined properly  
ex.  $y = \text{sgn}(x)$ , then  $e = y - x = \text{sgn}(x) - x$

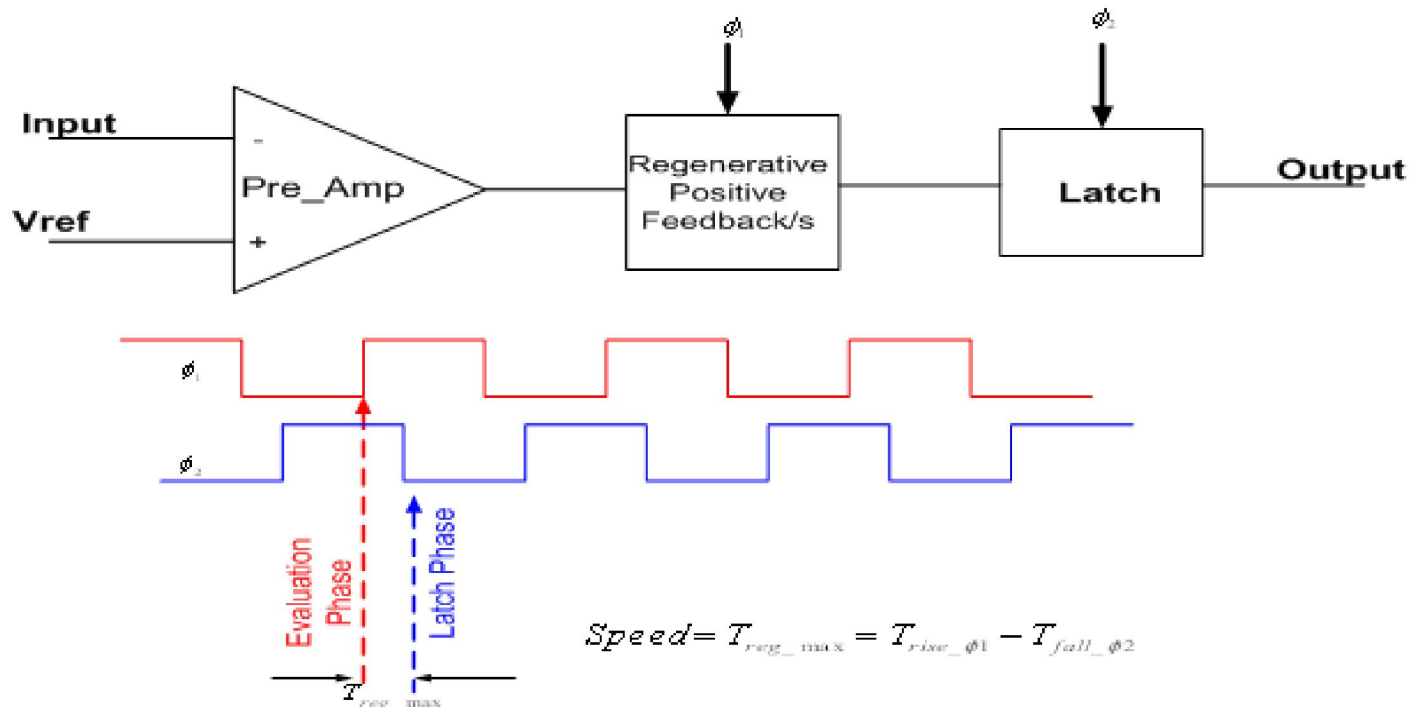
becomes an approximation when we  
claim noise is independent

Important model in  
Sigma Delta ADCs



- ❑ **Sampled Comparator:**
  - ❑ Much faster
  - ❑ clamp high gain stages
  - ❑ Need clock or digital signal

Concept:



## Errors in Comparators:

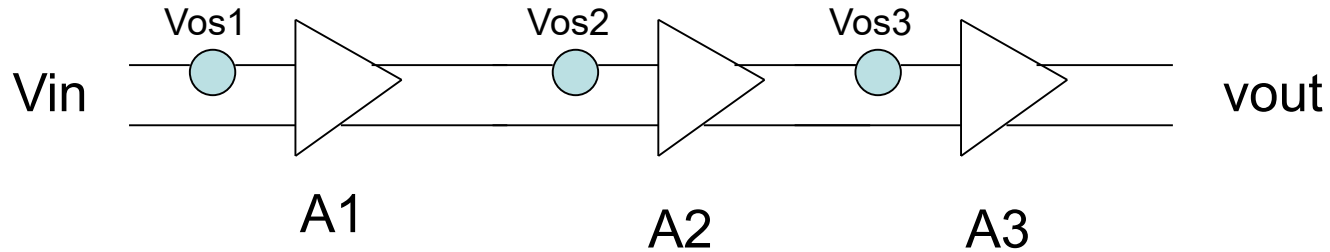
- Gain
- Offset
- Meta State
- Speed
- Kick Back
- Noises and supply noise

- **Input Offset** – It is the voltage that must be applied to the input to obtain the crossing point between low and high logic level
- $V_{out} = A \times V_{indiff}$  ,  $v_{ss} < V_{out} < V_{dd}$   $A=1000$  , time is also a factor
- **Sensitivity**- It is the minimum voltage or current that produces a consistent output signal within the expected comparison time- **Meta state**
- **Comparator response time** – It is minimum time interval required to achieve the proper logic output as a response to the minimum input step
- **Overdrive recovery time** – When the input signal is pretty large the gain stage saturates to the positive or negative rails quickly. If the input stage become small, the gain stage takes some time to react and generates the voltage required to produce the output voltage
- **Kick back input noise** – Caused in evaluation state due to transition response: **Switching noise**

# Offset and Gain – Basics in Comparator



$V_{os}(total)$



$$V_{os (total)} = \sqrt{V_{os1}^2 + \frac{V_{os2}^2}{A_1^2} + \frac{V_{os3}^2}{A_1^2 A_2^2}}$$

**Key:**

- $V_{os}$  is a statistic parameter
- Further stages (A2 and A3) have larger offset using minimum sizes..

## Example: 8 bit ADC

for 8 bit with  $V_{fs}=0.5v$  ,  $V_{dd}=1v$  at  $1/10LSB =$

$A1 \times A2 \times A3$  required  $\sim 5000 \rightarrow (1v/(0.5/255 \times 10))$

If:

$V_{os1}=2mv$ ,  $vos2=10mv$ ,  $vos3=20mv$

If we take:  $A1=10mv$   $A2=25mv$   $A3=20mv$

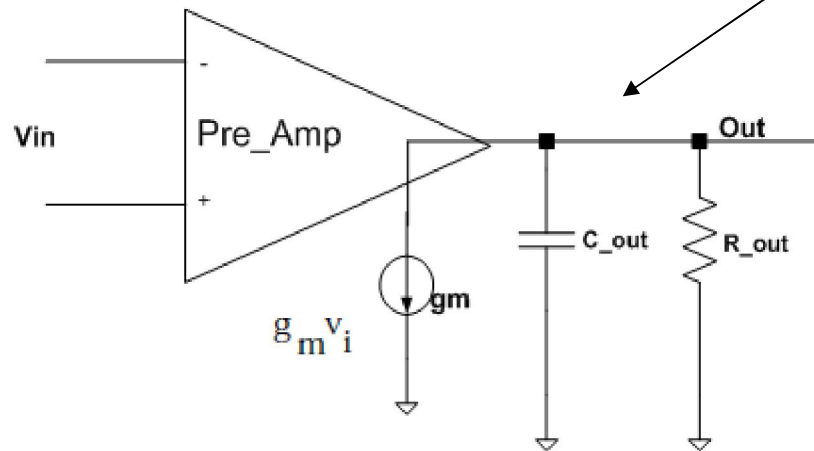
$$V_{os (total)} = \sqrt{2^2 + \frac{10^2}{10^2} + \frac{20^2}{10^2 25^2}} = 2.23mV$$

**Gm Rout ... diff stage..**

# Basic Analysis – Non Sampled – Non Latched



Model (vi is Vin)



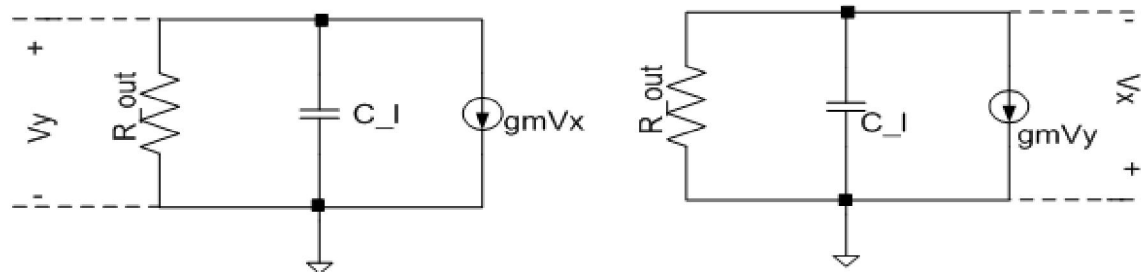
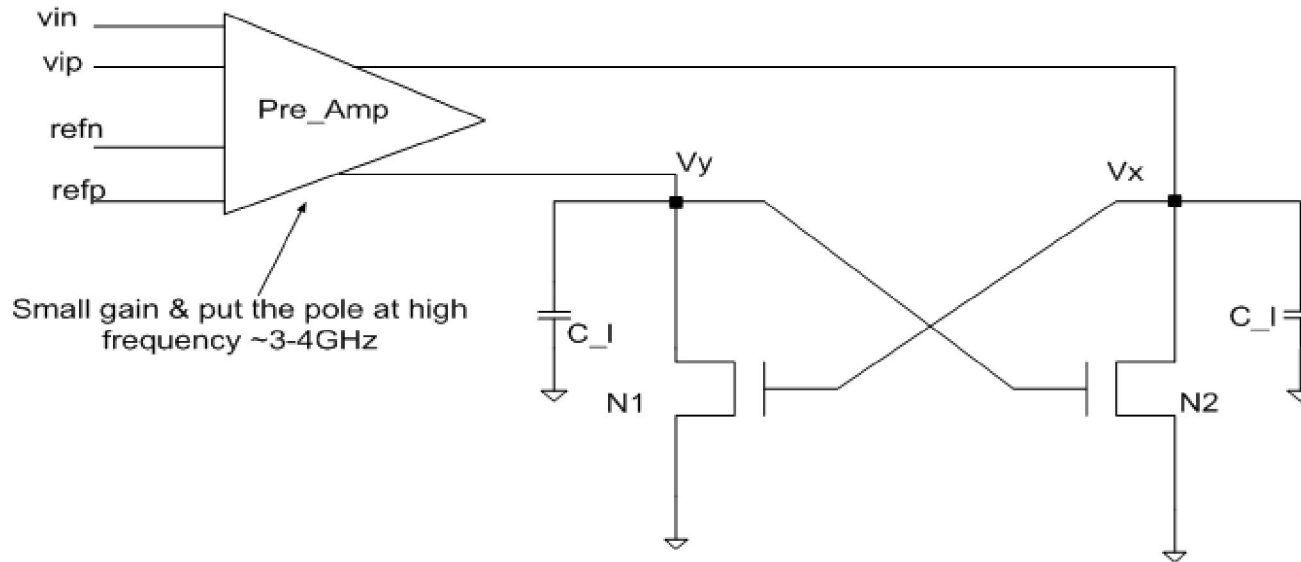
$$V_{out}(t) = V_{in} \cdot g_m \cdot R_{out} \cdot (1 - e^{-t_{reg}/\tau})$$

Where:  $\tau = R_{out} \cdot C_{out}$

$$g_m = \frac{I}{V_{in}}$$



# With Positive Feedback – Sampled



Small signal representation

# Math – Positive Feedback Comparators Latch



$$g_m \cdot V_y = -C_L \cdot \frac{dV_x}{dt} - \frac{V_x}{R_{out}} \quad g_m \cdot V_x = -C_L \cdot \frac{dV_y}{dt} - \frac{V_y}{R_{out}}$$

$$\begin{cases} \tau \cdot \frac{dV_x}{dt} + V_x = -A \cdot V_y \\ \tau \cdot \frac{dV_y}{dt} + V_y = -A \cdot V_x \end{cases}$$

→ where

$$\tau = R_{out} \cdot C_L$$

$$A = g_m \cdot R_{out}$$

Look not R but gm—  
Great speed improvement

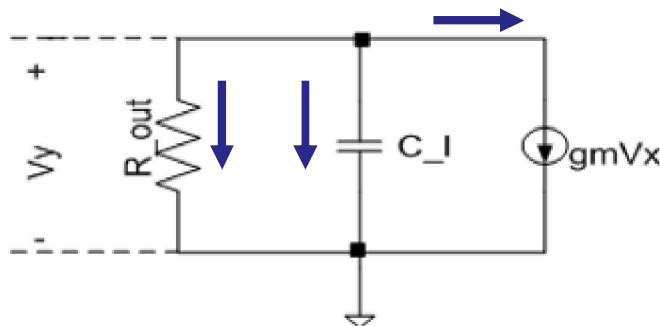
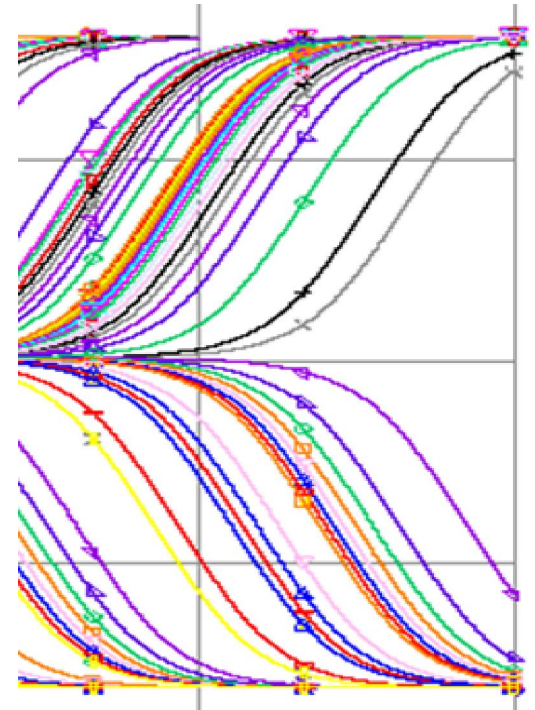
$$\tau \cdot \frac{d(\Delta V)}{dt} = \Delta V(A-1)$$

Solution →

$$\frac{\tau}{A-1} \cdot \frac{d(\Delta V)}{dt} = \Delta V$$

$$\Delta V = \Delta V_0 \cdot e^{-\frac{A-1}{\tau} t} \approx \Delta V_0 \cdot e^{-\frac{g_m}{C_L} t}$$

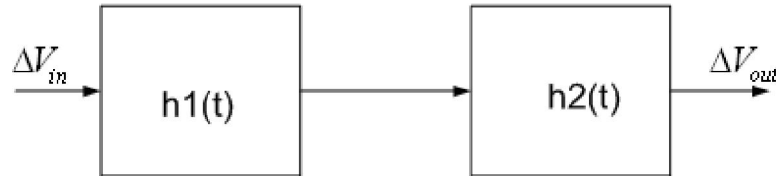
$$\frac{A-1}{\tau} = \frac{g_m \cdot R_{out} - 1}{R_{out} \cdot C_L} \approx \frac{g_m}{C_L}$$



# Cascade Stages

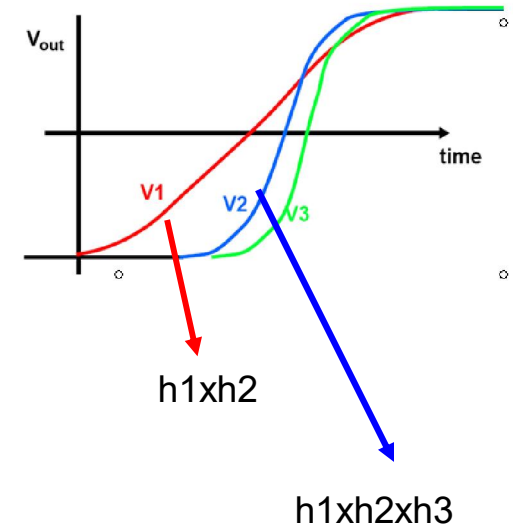


## Band width reduction



$$h(t) = h_1(t) * h_2(t) = \left( e^{\frac{g_m \cdot t}{C_L}} * e^{\frac{g_m \cdot t}{C_L}} \right) = t \cdot e^{\frac{g_m \cdot t}{C_L}}$$

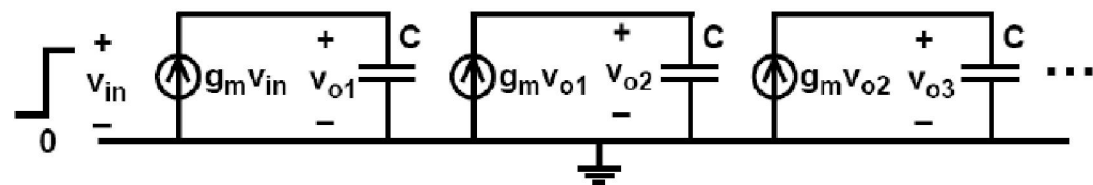
$$\Delta V_{out}(t) \approx \Delta V_{in} \cdot K \cdot t \cdot e^{\frac{g_m \cdot t}{C_L}}$$



Note: if we have n-stages of the regenerative feedbacks then

$$h_n(t) = h_1(t) * h_2(t) \dots * h_{n-1} * h_n = \left( e^{\frac{g_m \cdot t}{C_L}} * e^{\frac{g_m \cdot t}{C_L}} \dots * e^{\frac{g_m \cdot t}{C_L}} * e^{\frac{g_m \cdot t}{C_L}} \right) = \frac{t^{n-1} \cdot e^{\frac{g_m \cdot t}{C_L}}}{(n-1)!}$$

$$v_{oN} = \frac{\omega_u^N}{s^N} v_{in}$$

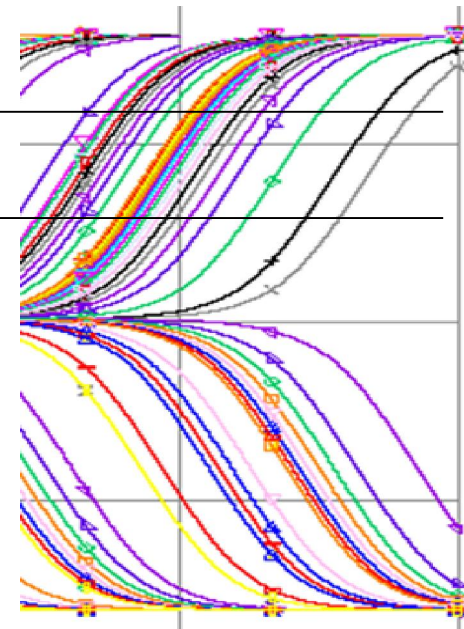


## Meta Stability in Comparator



- Meta-stability is a potentially catastrophic event that can occur when asynchronous inputs and regenerative/flip-flops are used
- Meta-stable outputs are not logic high or logic low and cause delays and system failures
- Meta-Stability is a probabilistic event, because the difference between the input signal and the reference voltage is a random variable
- The smaller the difference between the input signal and reference voltage, the longer the decision time required. On the limit the decision time can approach infinity

Not defined place





## Analytical Derivation of Meta-Stability & Mean Time To Failure (MTF)

- The probability event given following: where  $t$  is the actual comparison time,  $T$  is the allowed time

$$\Pr(t > T) = e^{\left(-\frac{A \cdot T}{\tau}\right)} = e^{-\frac{g_m \cdot T}{C_{out}}}$$

- If one can have a collection of  $N$  such comparators all clocking at a frequency  $f_s$ , then one can found MTF following

$$MTF \approx \frac{e^{\frac{A \cdot T}{\tau}}}{N \cdot f_s} = \frac{e^{\frac{g_m \cdot T}{C_{out}}}}{N \cdot f_s}$$

~ Set it for > month

- Example: 6-bit, 500MHz Flash ADC,  $T_{\max} = T_s/2 = 1\text{ns}$ ,  
 $\tau_u = 1/(2\pi \cdot 5\text{GHz}) = 32\text{ps}$ ,  $A_v = 3$ ,  $V_{FS} = 0.5V_{DD}$

$$P(\text{Error}) = \frac{2}{3} (2^6 - 1) \cdot e^{-1000/32} \cong 10^{-12}$$

- Mean time to failure (MTF)

$$MTF = \frac{1}{P(\text{Error}) \cdot f_s} = \frac{1}{10^{-12} \cdot 0.5 \cdot 10^9} \text{ s} = 2000 \text{ s} \cong 33 \text{ minutes}$$

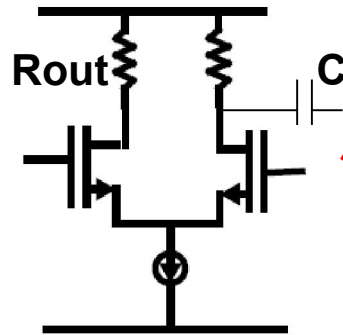
source: Prof. Murmann stanford

# Front Stage Architectures- with Resistors



## 1. Use small values of $R_l$

- Minimizes swing
- Shortens time constant

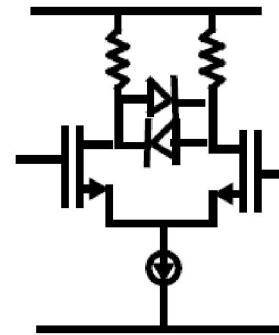


$$V_{out}(t) = V_{in} \cdot g_m \cdot R_{out} \cdot (1 - e^{-t_{reg}/\tau})$$

Where:  $\tau = R_{out} \cdot C_{out}$

## 2. Use Passive Clamps

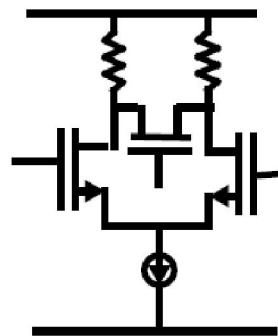
- Limits Swing
- Adds parasitics



$$BW = 1 / 2\pi(R_{out}C)$$

## 3. Use active nulling clamps

- Good in principle
- Tough clock generation problem



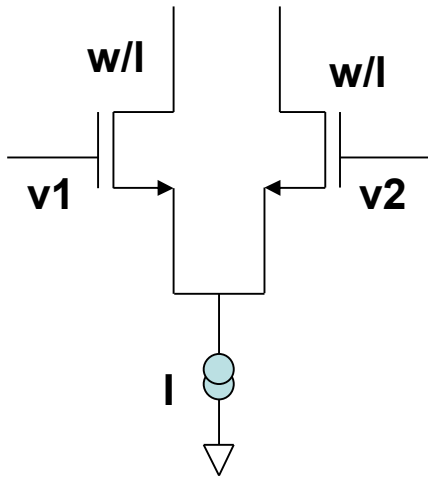
Source: P Gray

Summary:

Number of stages and gain needed is process and project dependent no fixed analytical solution.

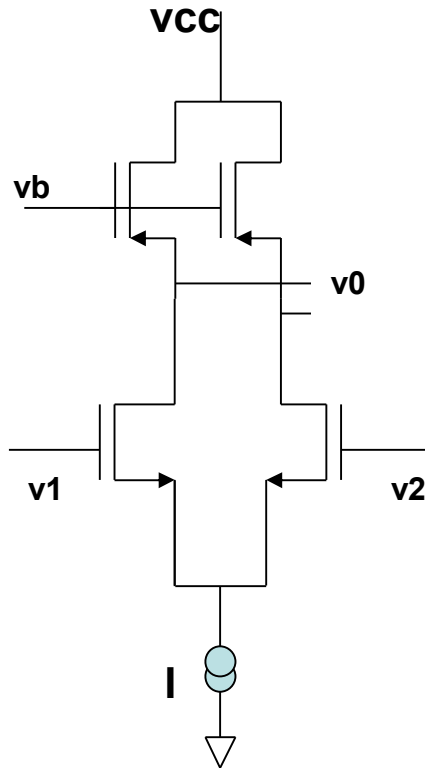
Best empirical  $g_m R = 4-8$  number of stages  $\sim 2-4$

# Input Stage: Architectures Options - Using PMOS



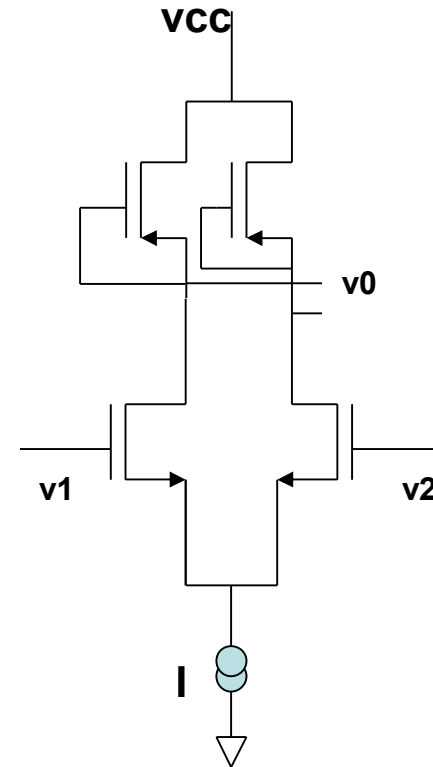
$g_m \times \text{what}$

A



$g_m \times R_{op}/R_{on}$   
Kickback  
Large gain

B

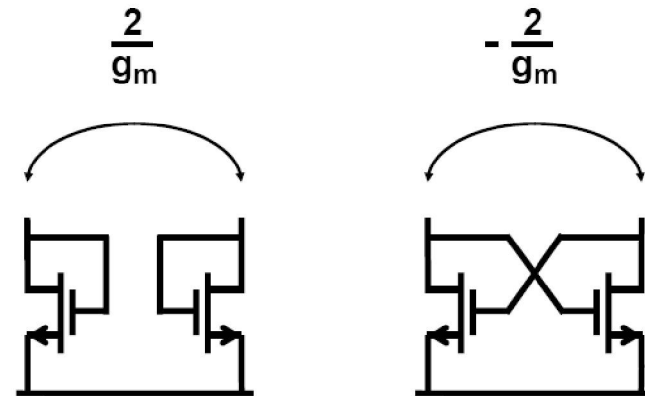
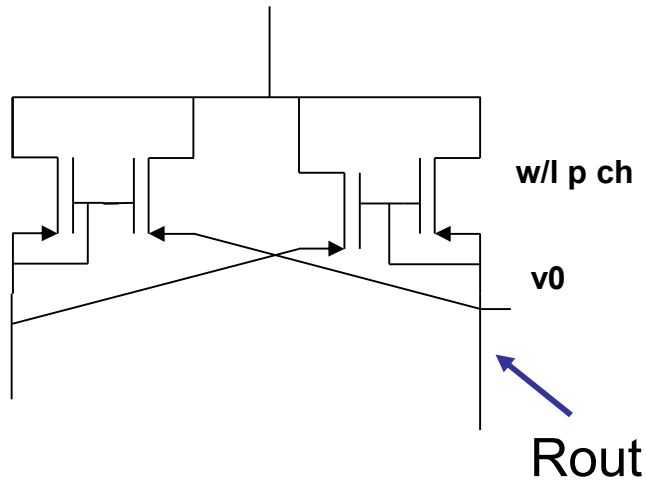


$g_m \times 1/g_{mp}$   
Low gain

C

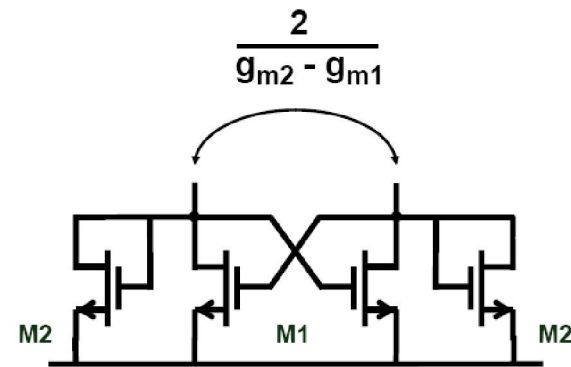


# Added Feedback to Load



positive feedback to raise Rout

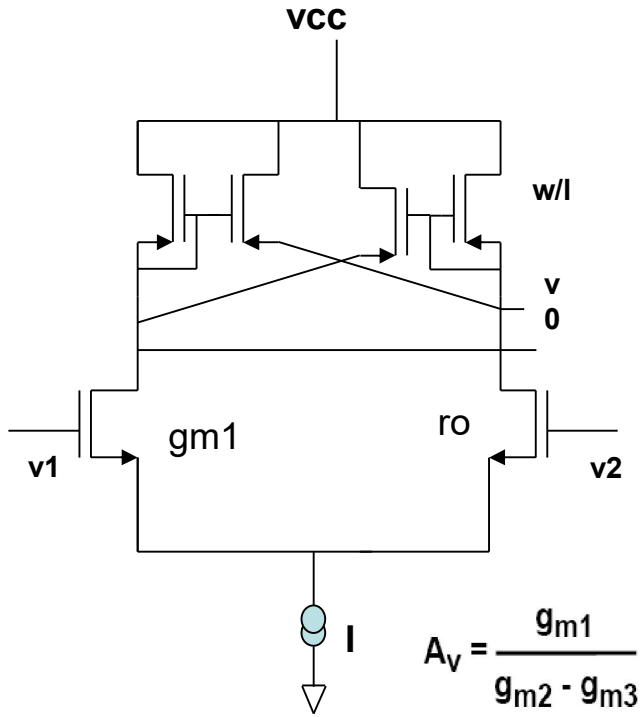
Effective rout =infinite



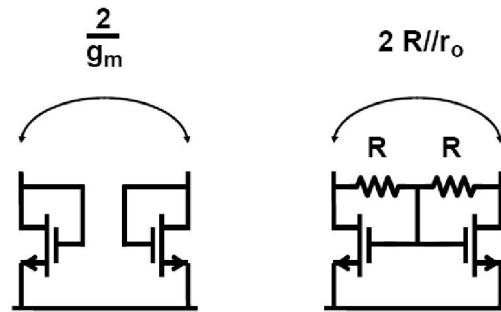
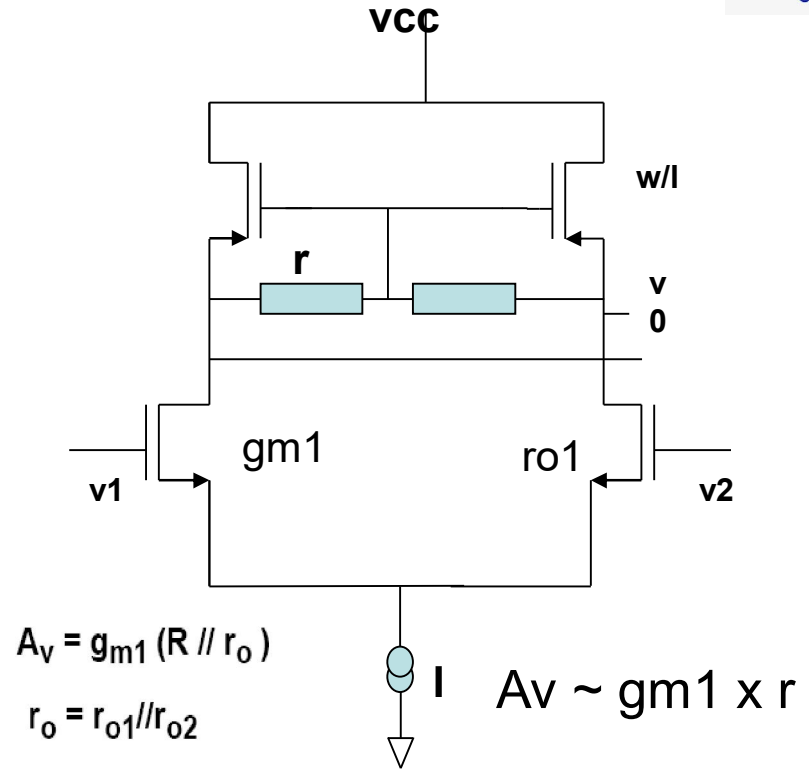
Values close to ∞ !

Source: Willy Snasen 2005

# Input Stage: Architectures Options



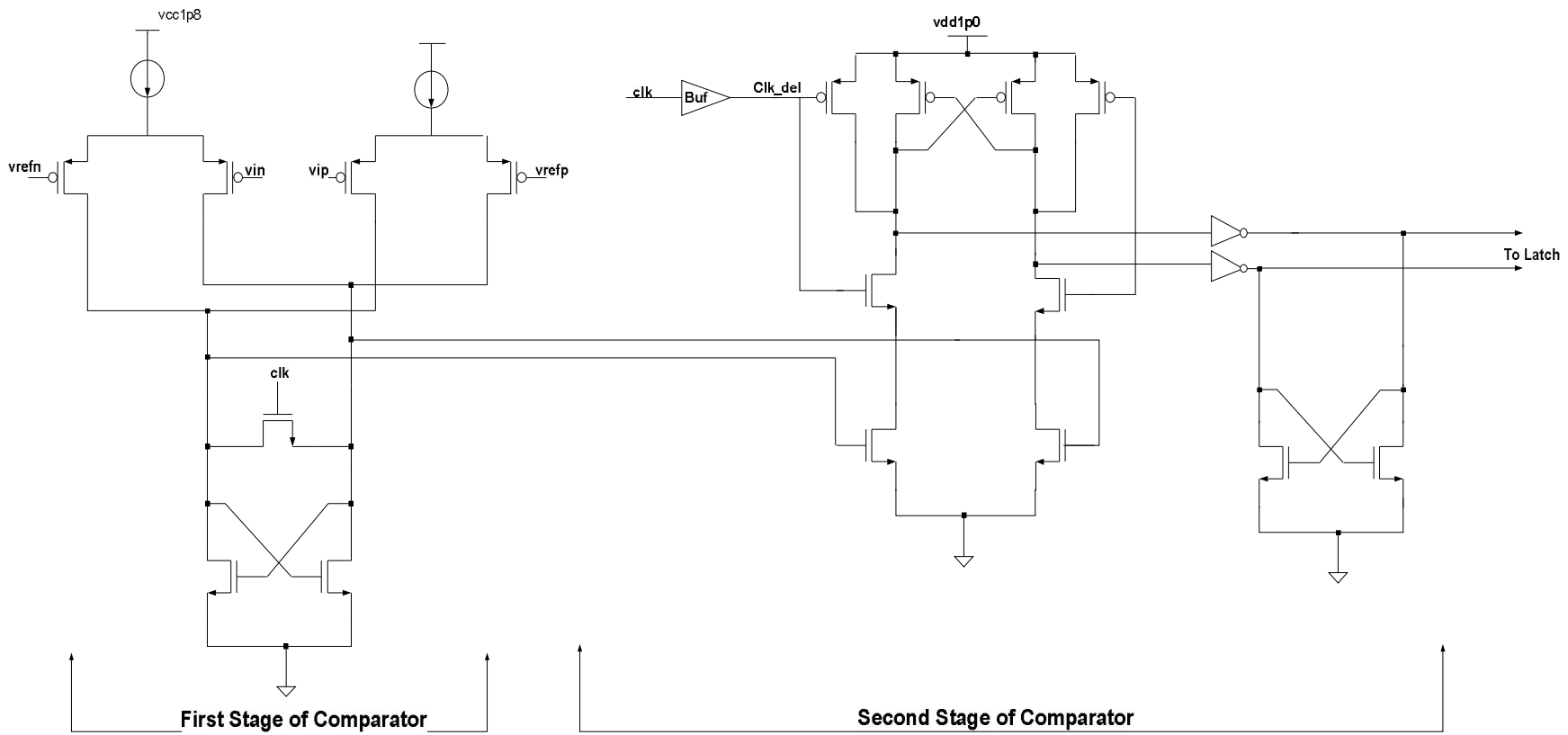
$AV \sim gm1 \times ro$



# *Comparator Examples*



# Differential Input Two Stage Comparator Option (no pre\_amp)



# More Example of comparators

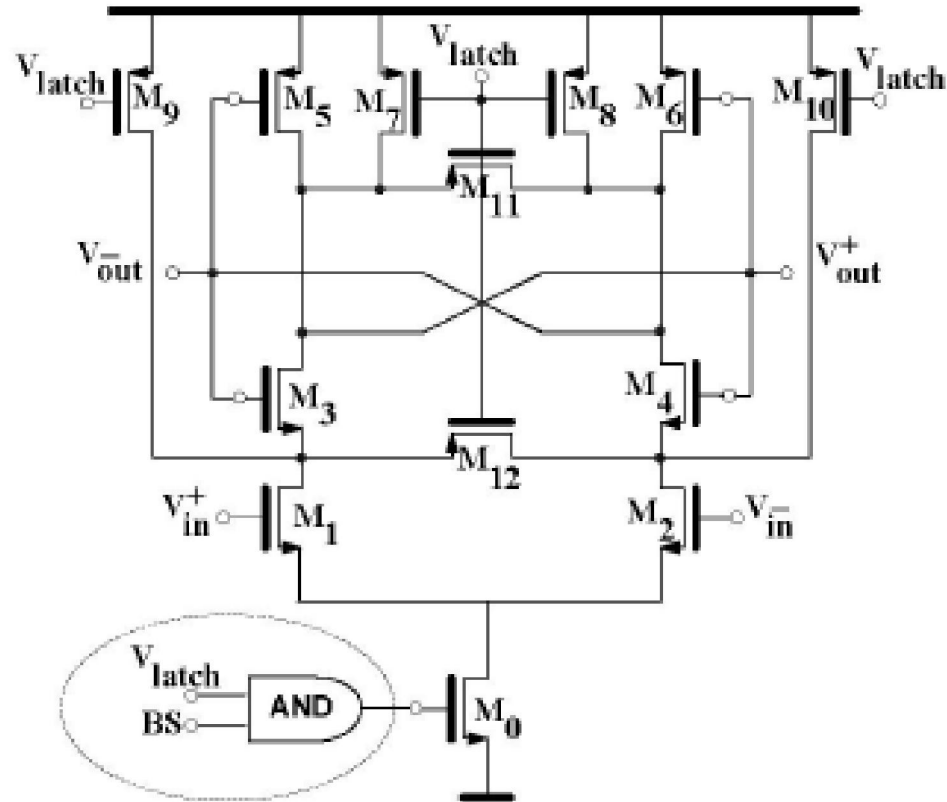
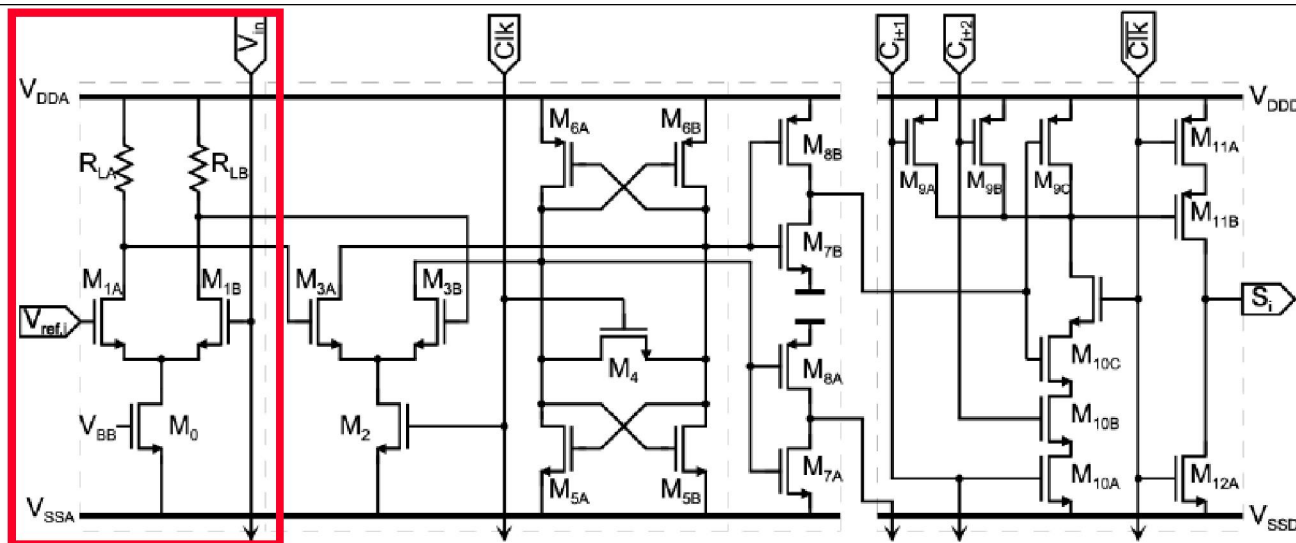


Fig. 5: High speed comparator and its switching circuit

A Power-Efficient 1.056 GS/s Resolution-Switchable  
5-bit/6-bit Flash ADC for UWB Applications

Jun-Xia Ma, Sai-Weng Sin, Seng-Pan U<sup>1</sup>, R.P.Martins<sup>2</sup>

## Special Topics – Calibrated and differential Architectures examples



- ◆ **Pre-amplifier [M1a,M1b]**
  - Differential Pair with resistive load
  - Reduce Kick-back noise
  - Reduce Input Referred Offset Comparator

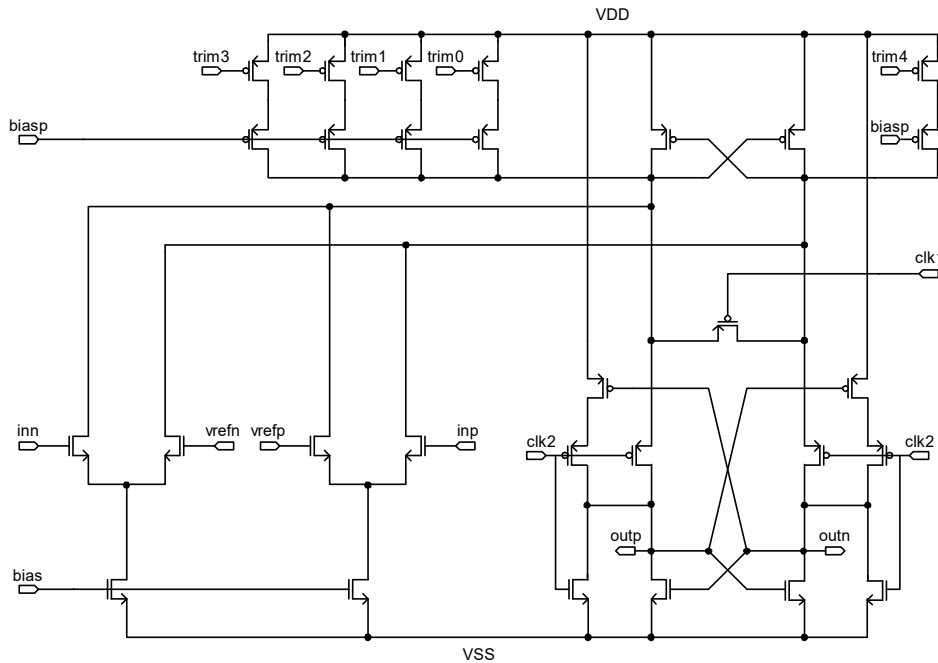
Source: Esat-Micas esscirc00

## **Offset Calibration of Comparators:**

**Why? → Main reason lower input capacitance**

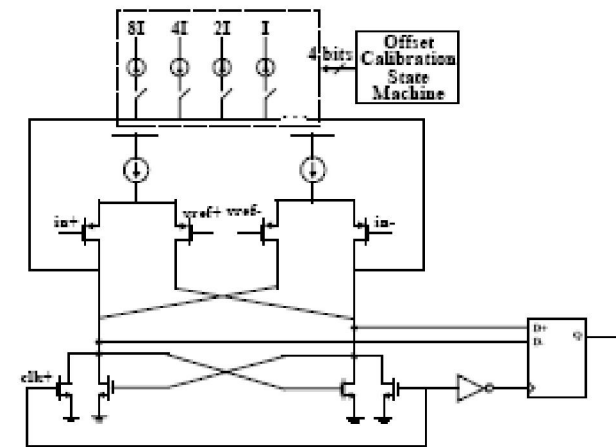


# Differential – Latched with Calibration Differential



## A 10.3GS/s 6bit (5.1 ENOB at Nyquist) Time-Interleaved/Pipelined ADC Using Open-Loop Amplifiers and Digital Calibration in 90nm CMOS

Ali Nazemi<sup>1</sup>, Carl Grace<sup>1</sup>, Lanny Lewyn<sup>1</sup>, Bilal Kobeissy<sup>1</sup>, Oscar Agazzi<sup>1,2</sup>, Paul Voois<sup>1</sup>, Cindra Abidin<sup>1</sup>, George Eaton<sup>1</sup>, Mahyar Kargar<sup>1</sup>, Cesar Marquez<sup>1</sup>, Sumant Ramprasad<sup>1</sup>, Federico Bollo<sup>2</sup>, Vladimir A. Posse<sup>1</sup>, Stephen Wang<sup>1</sup>, Georgios Asmanis<sup>1</sup>  
 ClariPhy Communications, Inc., 16 Technology Drive, Suite 165, Irvine, CA 92618, USA, E-mail: [ali.nazemi@clariPHY.com](mailto:ali.nazemi@clariPHY.com)  
<sup>2</sup>ClariPhy Argentina S.A., Cordoba, Argentina



Comparator schematic with digital offset calibration.

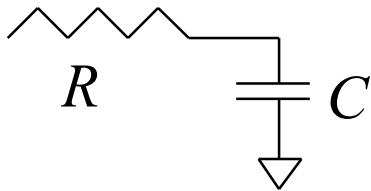
# *End Lecture 8*

## *Additions*

# Basic Feedback to Boost BW



“SLOW”

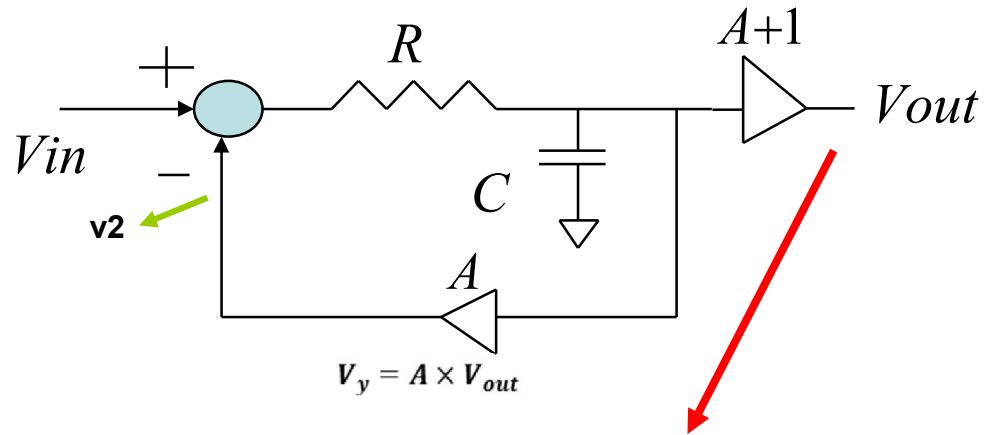


$$V_{out}(t) = V_{in}[1 - e^{-t/\tau}]$$

$$\tau = RC$$

$$Gain_{DC} = 1$$

“FAST”



$$V_{out}(t) = V_{in}[1 - e^{-t(A+1)/\tau}]$$

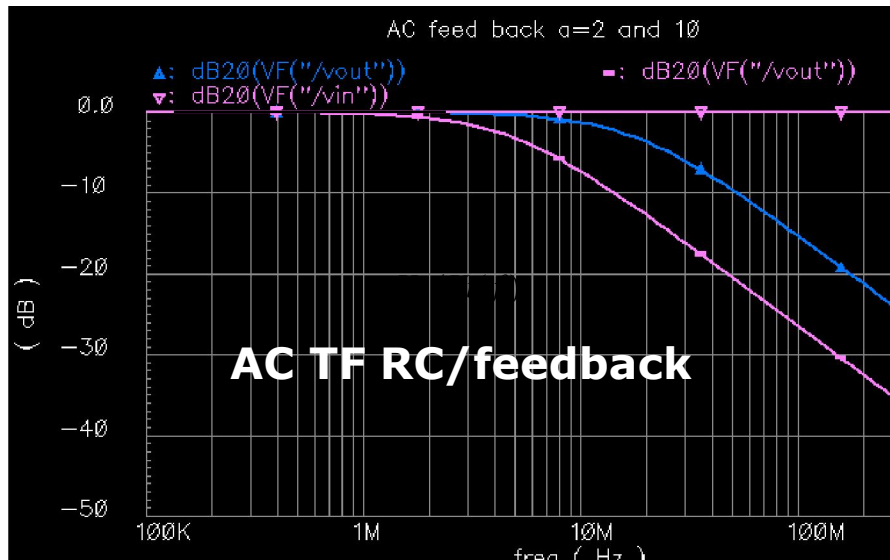
$$\tau(\text{new}) = RC/(A + 1)$$

$$Gain_{DC} = 1/(1 + A)$$

But at  $V_2$ :

$$Gain_{DC} = \frac{A}{1 + A} \approx 1$$

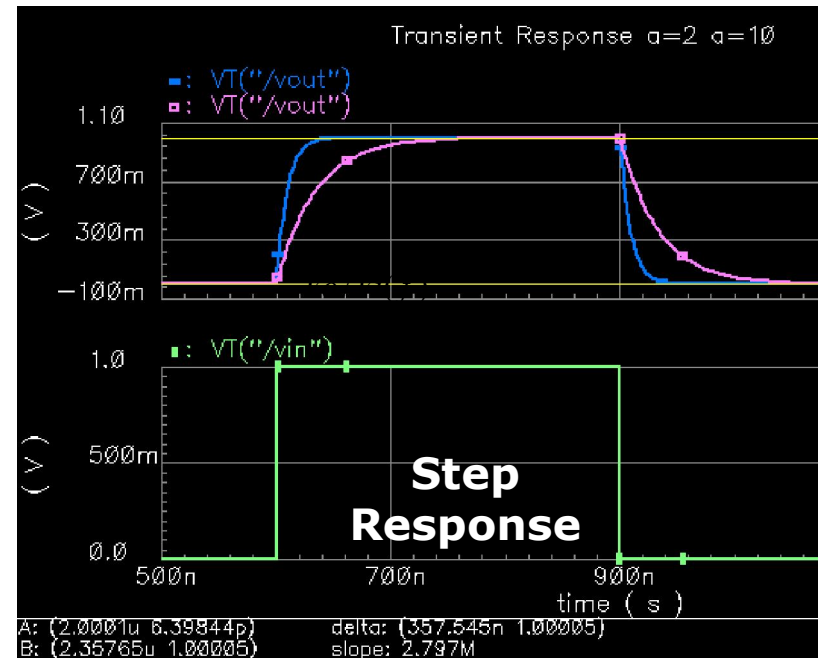
# Simulation: A=2 and A=10



Frequency

$$V_o / V_i(f)$$

AC Response



Time

Time Domain