



Welcome to
046188 Winter semester 2012
Mixed Signal Electronic Circuits
Instructor: Dr. M. Moyal

Lecture 04

Jitter and Noise in Mixed Signal (Converters, and analog blocks).

1. Jitter + few examples (summary lect1-3)
2. Noise sources in Mixed Signal
3. Passives elements in Silicon

www.gigalogchip.com



number of bits n	S/N Accurate dB	S/N n 6.02 + 1.76 dB
1	6.31	7.78
2	13.30	13.80
3	19.52	19.82
4	25.59	25.84
5	31.65	31.86
6	37.70	37.88
7	43.76	43.90
8	49.82	49.92
9	55.87	55.94
10	61.93	61.96

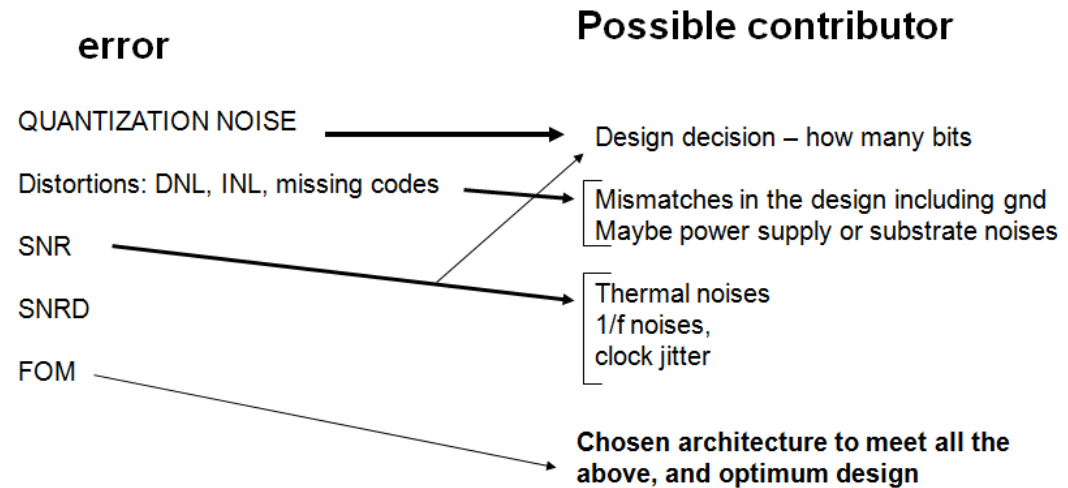
Table 1.1: S/N as a function of the number of bits n

The signal-to-noise of an n-bit converter is accurately modeled with:

$$S/N(n) = \frac{A_1}{A_{\text{quantization}}} = \frac{2^{n-1} + \sum_{m=1}^{\infty} \frac{2}{m\pi} J_1(2m\pi 2^{n-1})}{\sqrt{\sum_{q=1}^{\infty} (\sum_{m=1}^{\infty} \frac{2}{m\pi} J_{2q+1}(2m\pi 2^{n-1}))^2}} \quad (1.45)$$

$$ENOB(\text{bit}) \equiv \frac{SNDR(\text{effective}) - 1.76}{6.02}$$

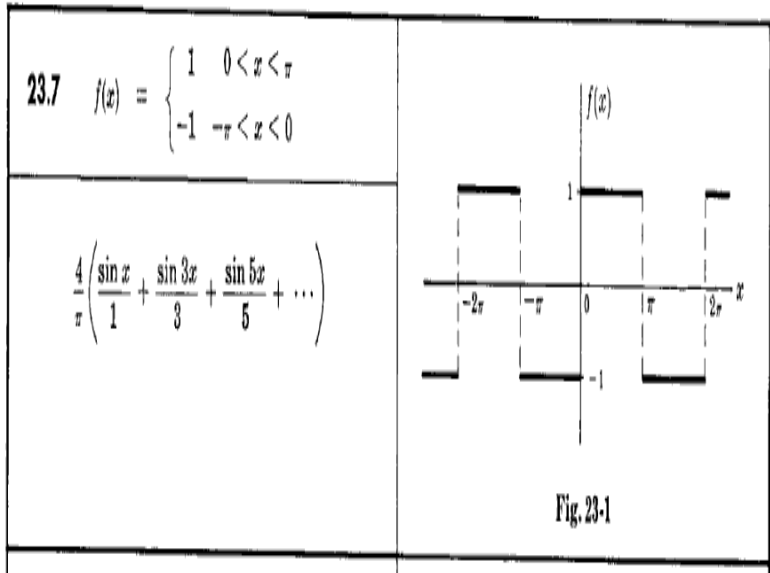
Industry "Accepted" Formula more correct for >3 bits..



DNL Definition
Differences between two adjacent output digital or analog compared to a step size of LSB weight.



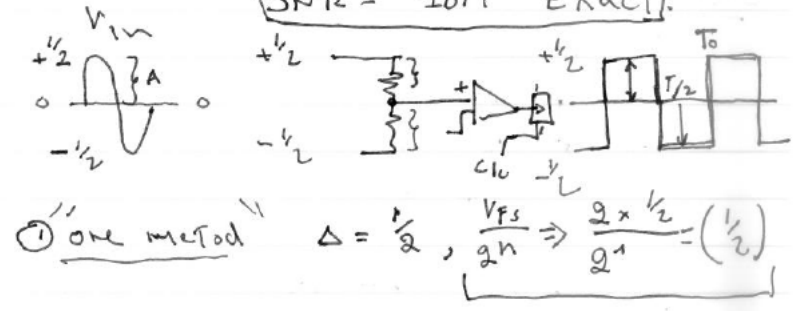
In class derivation example..exact SNR 1-bit



6.28dB is the correct answer not 7.78

Example - 046188

SNR - 1bit EXACT



$$SNR \Rightarrow \frac{\frac{A^2}{2}}{\frac{B_n}{12}} = 20 \lg \frac{\frac{A}{\sqrt{2}}}{\frac{B}{\sqrt{12}}} \text{ OR } 6.02n + 1.76 \text{ dB}$$

$$= 6.02 + 1.76 = 7.78$$

EXACT $SNR = \frac{P_0(\text{signal})}{P_0(\text{noise})}$

$\leq \sqrt{\left(\frac{2}{\pi^2}\right)} = 0.450V$

$P_0(\text{signal}) \Rightarrow$ FFT

$$P_{0 \text{ TOTAL}} = \frac{1}{T_0} \int x^2 dx = \frac{1}{T} \int_0^T \left(\frac{1}{2}\right)^2 dx = \frac{1}{4}$$

$$SNR = 10 \lg \frac{\left(\frac{2}{\pi^2}\right)}{\left(\frac{1}{4} - \frac{2}{\pi^2}\right)} = 10 \lg \frac{0.2025}{0.0475} = 6.28 \text{ dB}$$

In class derivation example..of FOM estimation

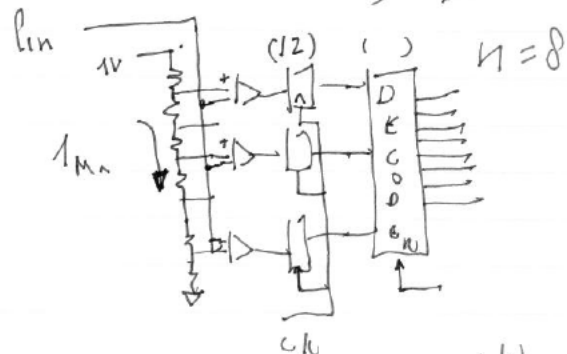


046188

(CALCULATE)
FOM EXAMP/1.

#2

8 bit FLASH, $f_{in} = 2\text{MHz}$



Given

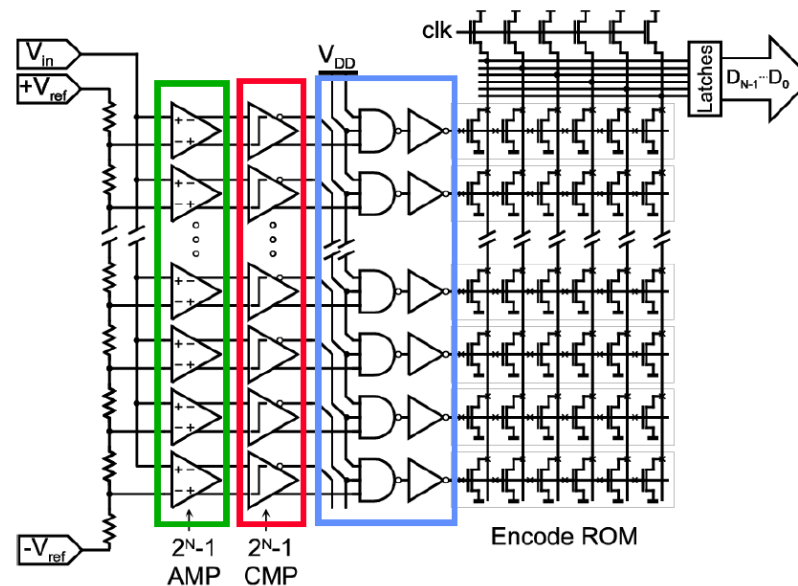
$$C_{mas} \Rightarrow 0.13\mu \quad P = 10 \frac{\text{hW}}{\text{gate} \cdot \text{MHz}} \quad \left. \begin{array}{l} \\ \text{and} \end{array} \right\}$$

$$P = (255 \times 100\text{mA}) + 1\text{mA} + \left(\frac{6}{12} + \frac{5}{12} \right) 255 \times 20\text{hW}$$

$$\sim 25.5 + 1 + 0.158 = 26.658\text{mA}$$

$$FOM = \frac{26.658 \cdot 10^{-3}}{2 \times 10^6 \cdot 2 \cdot 256} = 26 \cdot 10^{-12}$$

$$FOM = 26$$



Step1: Inventory: rough est.

- 255 comparators, 255 resistors
- 255 FFs ~3 gates x 255
- Thermometer/binary nand+inv+8trans ~3gates x 255
- Clock to comp ~ 5/line x 255

Key: MOSTLY COMP POWER Why 100Ua/COMP ?

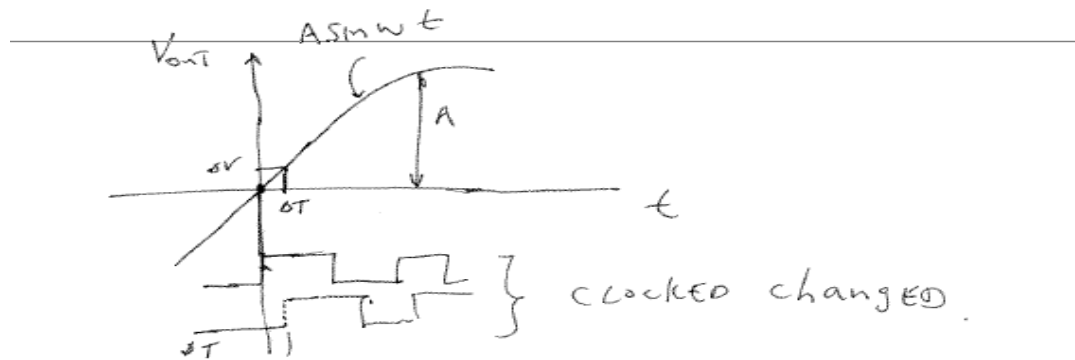


CLOCK JITTER EFFECT IN CONVERTERS

JITTER ERRORS- what will it do to a converter ?- Aparatus error



SAMPLING TIME UNCERTAINTY



$\Delta T = \text{JITTER}$

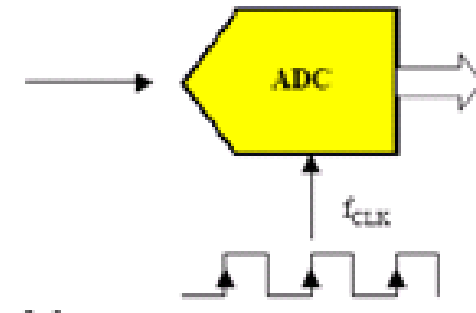
$A = \text{Signal amplitude Sine wave} \Rightarrow A \sin \omega t$

SEARCH FOR MAX ΔV BUT $\Delta V \leq 1 \text{ LSB}$

$$\left. \frac{\Delta V}{\Delta T} \right|_{\text{max}} = \left. \frac{dV}{dt} \right|_{t=\phi} = A\omega \cos \omega t = A\omega$$

BUT $2A = \text{CONVERTER RANGE} \Rightarrow \text{LSB} = \frac{2A}{2^n}$

FIND t_{JITTER} THAN NEVER COARSE TO GO OVER LSB





$$\frac{\Delta V}{\Delta T_{\max}} = A \omega$$

$$\Delta T_{\max} = \frac{Z_{SB} \cdot 2}{\omega \cdot g_n \cdot Z_{SB}} = \frac{1}{\pi f_{in} \cdot 2^n}$$

ΔT_{\max} = Peak to peak clock uncertainty

bits	ΔT_{\max}
10	31 ps
12	7.77 ps
14	1.9 ps
19	0.19 ps

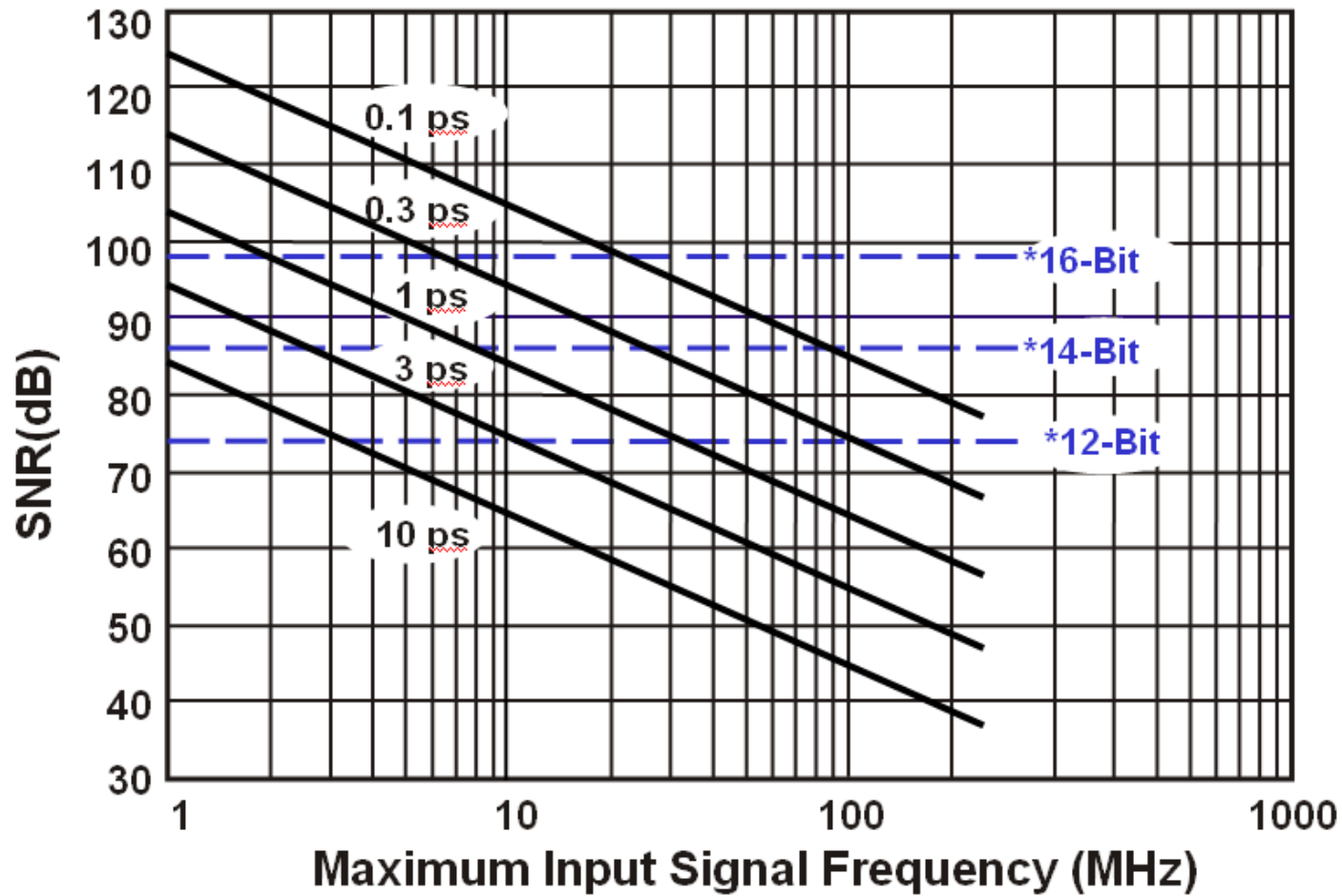
$f_{in} = 10 \text{ MHz}$
 400 MHz

Today PLL's $\Rightarrow \approx 2 - 20 \text{ ps}$ pll-pll

KEY: SNR due to jitter ifs input frequency dpepndent !



Jitter Limits



* Equivalent converter SNR



***It is clear that the Clock effect is on high changing input
(higher frequency..***

But..

It is unclear where in the frequency lies this error ?

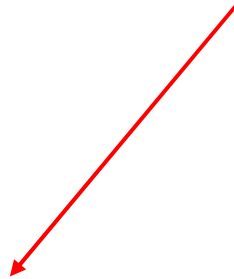
Random ?

How does it correlate to Phase noise ?

Part of the project is to plot spectrum of the Jitter..



Noise In Resistors and CMOS Transistors



Noise set the fundamental limits of performance

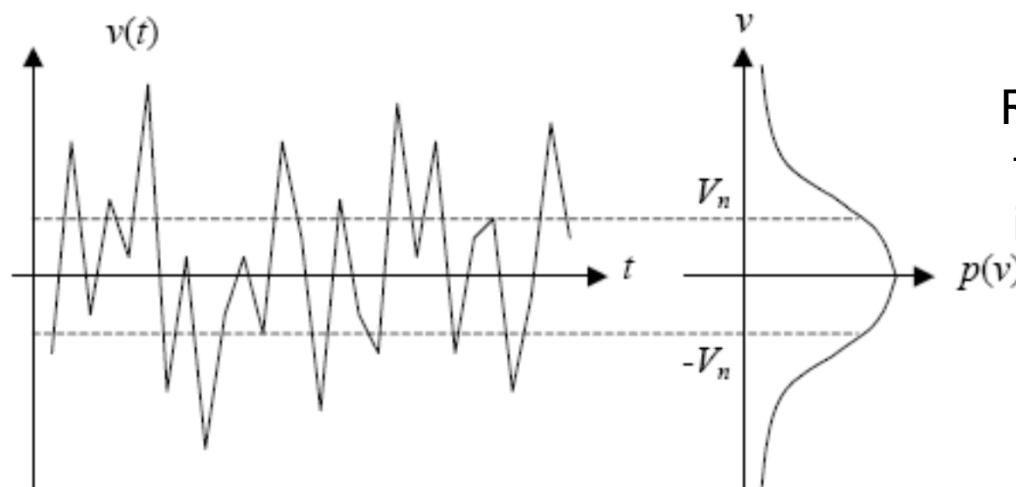
Basics of Noise



Thermal energy in the device causes random charge carrier fluctuations, called thermal noise.

Thermal noise is considered a “white” noise source because its power spectral density (PSD) is independent of frequency up to about f_t (or the BW of the device).

We observe Voltage or Current noise of time and construct the PDF function



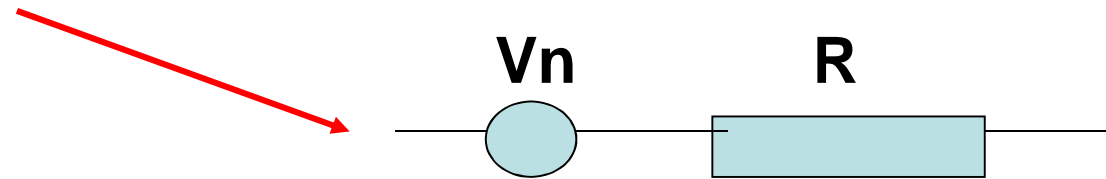
Root mean square is a way to quantify noise and as density in nV (mV) per square root Hz.

Source: T.H.Lee

Example: A resistor thermal noise



MODEL



Take $kT = 26\text{mV} \times 1.6\text{e-}19\text{C} = 41.6 \text{ e-}22 \text{ v square /ohm Hz}$

The model is a resistor with a voltage source

$4kTR$ units are V^2/Hz

Can Take the square root – noise density S_n)

$$V_{noise}^2 = 4 \cdot k \cdot T \cdot R \cdot \Delta f$$

$$1K\Omega \Rightarrow 4.09 \cdot (\eta V / \sqrt{Hz})$$



NOISE EFFECT IN CMOS TRANSISTORS



For MOS, We define 2 type of noises,

Two noise mechanism:

Thermal noise – white noise up to ft of the device

Flicker noise – low frequency noise (not so low for Nmos)

It possible that in one mode or another $1/f$ is almost nulled

There exists other type of definitions for noise – Phase Noise, Electron Noise (ENC)

Which are all derivatives or impacted from the basic Flicker and thermal noise (Mixing)



Thermal noise: Thermal energy in the device causes random charge carrier fluctuations, called thermal noise.

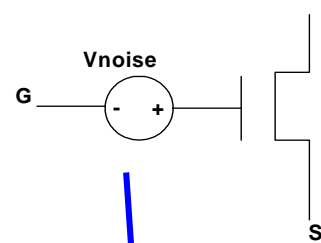
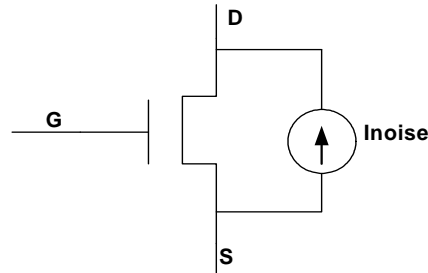
Thermal noise is considered a “white” noise source because its power spectral density (PSD) is independent of frequency up to about f_t (or the BW of the device).

Flicker noise: results from random motion of charge over potential barrier (need a gate).

Transistor Thermal and 1/f Noise



Gamma a process number ~1-2)



on class derive I noise

$$I_{noise}^2 = (2/3)4 \cdot k \cdot T \cdot \gamma \cdot g_m \cdot \Delta f$$

$$V_{in_noise}^2 = \frac{(2/3) \cdot 4 \cdot k \cdot T \cdot \gamma \cdot \Delta f}{g_m}$$

Thermal

$$I_{noise}^2 = \frac{K_0 \cdot k \cdot T \cdot g_m \cdot g_m}{W \cdot L \cdot f} \cdot \Delta f \sim \frac{1}{L \cdot L \cdot f} \cdot \Delta f$$

function of L² not W

$$V_{in_noise}^2 = \frac{K_0 \cdot k \cdot T}{W \cdot L \cdot f} \cdot \Delta f$$

“1/f”

function of gm (sqrt W/L)

A model

For a voltage mode device (input referred noise)

For current mode device (a current source)

METAL OXIDE SEMICONDUCTOR FET (MOSFET)

The equivalent input noise voltage density of a metal oxide semiconductor FET (MOSFET) can be expressed as:

$$e_n^2 = 4kT \frac{2}{3g_m} + K_4 \frac{1}{f} \cdot \frac{1}{W \cdot L}$$

Sum it up ! In nch
1/f corner ~50-500Hz

Noise and frequency



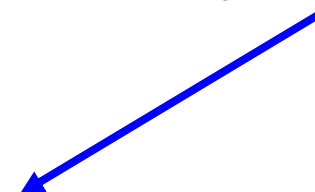
$$V_{in_noise}^2 = \frac{(2/3) \cdot 4 \cdot k \cdot T \cdot \gamma}{g_m} \cdot \Delta f$$

The total noise (input referred) is always the area under the density noise curve the integral from $f=0$ to f ="your design system uses"- hard to tell

Noise is not always flat in all frequency

Example: the total thermal noise from 1KHz-10MHz noise is:

$$V_{total\ 1-10\ Mhz_noise} = \sqrt{\frac{(2/3) \cdot 4 \cdot k \cdot T \cdot \gamma}{g_m}} \cdot \sqrt{9.999000}$$





as voltage mode:

Thermal noise:

$V_n^2 / \Delta f \approx 8 KT / 3 g_m$ use large w/l and g_m \rightarrow lower noise

1/f noise

$V_n^2 / \Delta f \approx K_o KT / f W L$ use large area $W \times L$ \rightarrow lower noise

Drain current noise

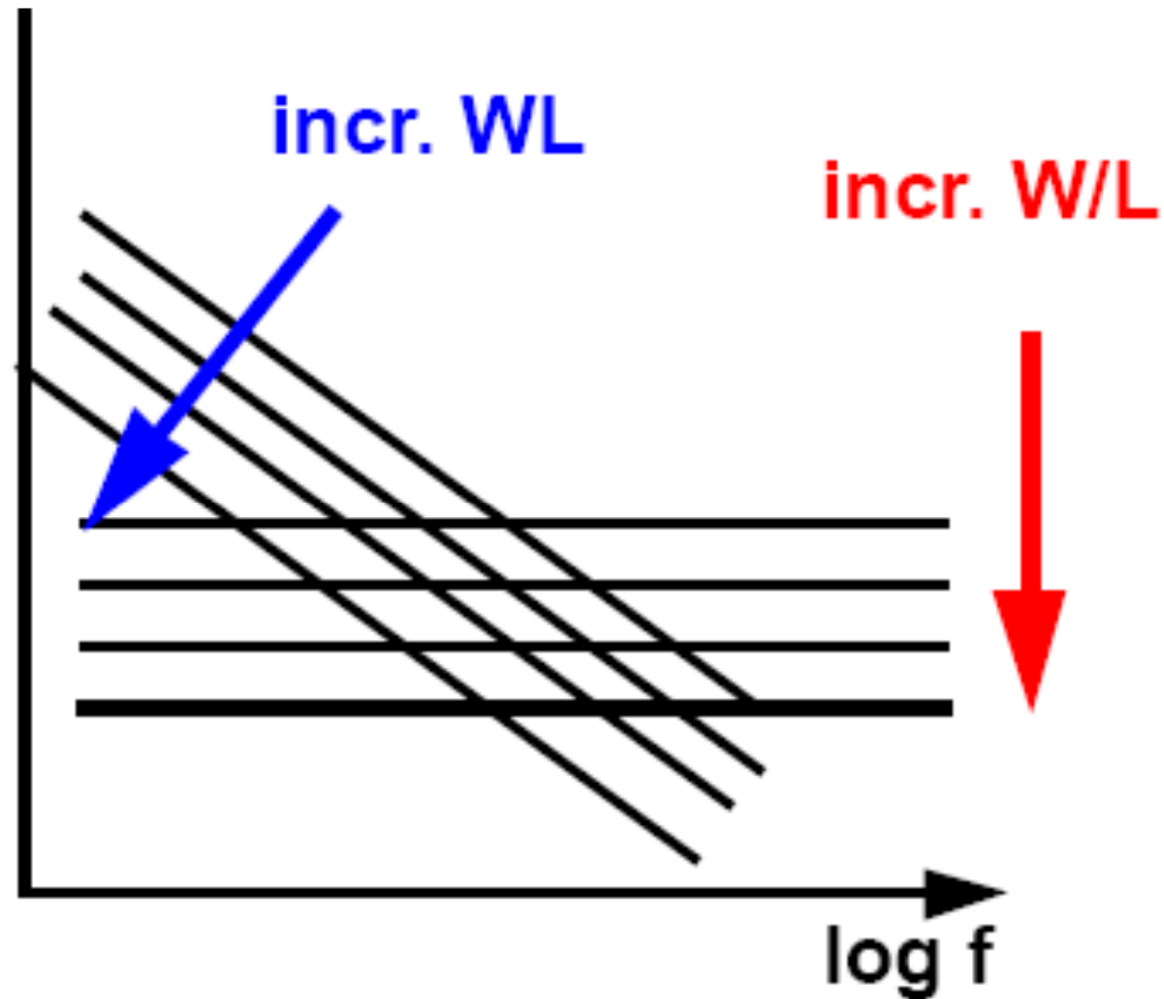
$\underline{I_n^2 / \Delta f} \approx 8 KT g_m / 3$ function of g_m ($\sqrt{W/L}$)

$\underline{I_n^2 / \Delta f} \approx \underline{K_o} KT g_m^2 / f W L$ function of L^2 not W

For fixed I_{ds} in strong inversion (Voltage mode)

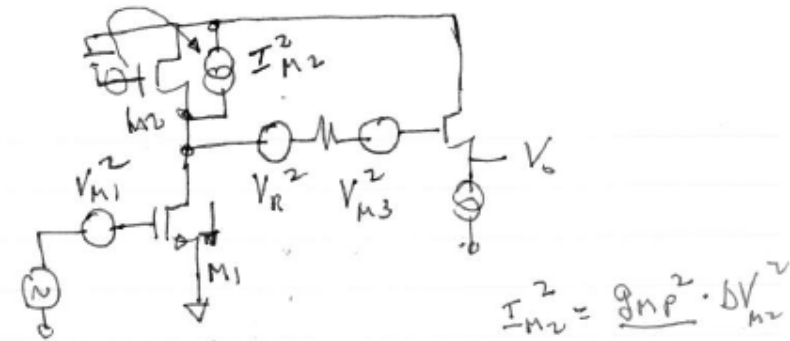
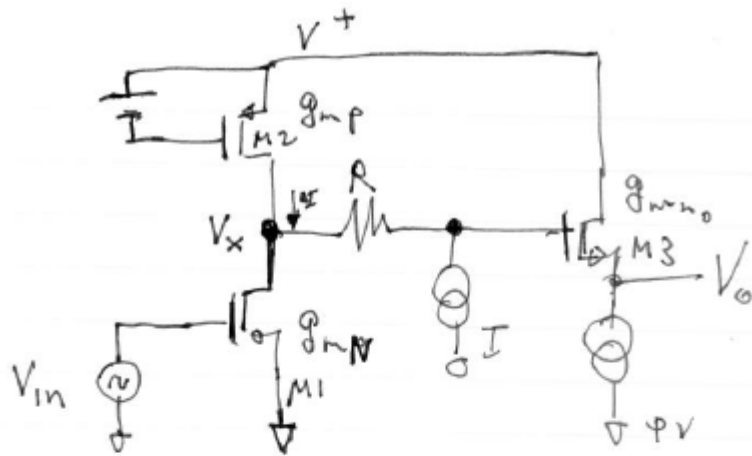


$$V / \sqrt{\text{Hz}}$$



flicker and thermal noises interactions

Example 1ckt noise of 2 gain stages – in class



$$I_{n2}^2 = g_{m2}^2 \cdot \Delta V_{n2}^2$$

Total 'input' noise

$$\sqrt{V_{n1}^2 + \frac{I_{n2}^2}{g_{m1}^2} + \frac{V_{n2}^2 + V_{n3}^2}{g_{m1}^2 \cdot (R_{oM1} || R_{oM2})^2}}$$

→ what TOTAL input referred noise AT moderate f (consider thermal noise)

→ effect/consideration for converter?

$$V_{n1}^2 = \frac{8}{3} \frac{kT}{g_{m1}}$$

$$V_{n2}^2 = \frac{8}{3} \frac{kT}{g_{m2}}$$

$$V_{nR}^2 = 4kT \cdot R$$

$$V_{n3}^2 = \frac{8}{3} \frac{kT}{g_{m3}}$$

$$g_m = \sqrt{k \frac{W}{L} I}$$

$$V_x = V_{in} g_{m1} \left(\frac{R_o || R_o}{g_{m2} R} \right)$$

$$\Delta I = g_{m1} \cdot \Delta V_{in}$$

$$V_n^2 = \sqrt{\frac{8}{3} \frac{kT}{g_{m1}} + \frac{\frac{8}{3} kT \sqrt{g_{m2}^2}}{g_{m1}^2 / g_{m2}} + \frac{4kTR + \frac{8}{3} kT / g_{m3}}{g_{m1}^2 \cdot (R_{oM1} || R_{oM2})^2}}$$

↑ LARGA
 MAKE $g_{m1} \uparrow$ W/ $L \downarrow$
 MAKE $g_{m2} -$ LOW \downarrow

FOR ADC DAC $f_{NL} \sim \frac{1}{2} \text{LSB} \Rightarrow \frac{V_{FS}}{2^n} \sim \frac{1}{2} V_2 \cdot (BW)^p$



Passive RC in Silicon

and **MISMATCHES**



Mismatches

**One of the most critical
parameters in Mixed signal design.**

**Mostly ignored with normal
simulators**

Si passive Elements Used in Mixed Signal



Silicon Resistors:

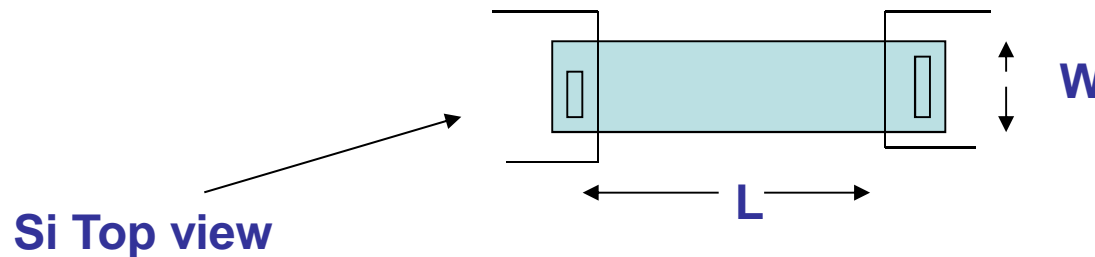
In Silicon $R = \text{Sheet resistance} \times \text{Number of square.}$
Made of Si doped material. (poly and N+ or P+ or both.)

Problems: They have parasitic capacitances, (BW limitation) and
Temperature variations :

$$R = v/i$$

$$R = R_s \cdot \text{squares} (1 + \alpha T / T_0)$$

$$R \propto 1 / (\mu q N a)$$





Polysilicon

Grown from pyrolytic decomposition of silane (SiH_4) at about 600°C .

The polycrystalline structure is made of monocrystal grains size in the range of $0.1 - 1 \mu\text{m}$.

The typical layer are $200 - 600 \text{ nm}$ thick with long term standard deviation in the 2% range.

The mobility is low because of the grain border resistance ($30-40 \text{ cm}^2/\text{Vs}$).

In order to have a low sheet resistance the polysilicon must be strongly doped ($10^{20}-10^{21} \text{ cm}^{-3}$). Part of the doping saturates the localized levels due to the grain border. The sheet resistance is in the range $20 - 40 \Omega/\square$.

The sheet resistance can be reduced by using sandwich layers (polysilicide) made of 200 nm of polysilicon covered with a film of refractory metal silicide (WSi_2 , MoSi_2 , TiSi_2). The sheet resistance is reduced to $1 - 5 \Omega/\square$.



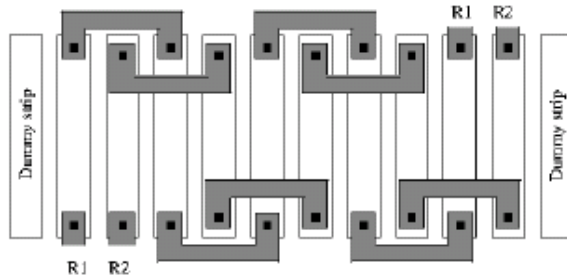
Common type of resistors in IC - silicon

RESISTOR	Range	0hm/sq Value	% (1x1uu) matching	ppm/c ,ppm/v temco
Poly				
No silicide	+/-15%	200-1k	2%	0-400 ppm/c
With silicide		6	-	-
N+/P+	+/-20%	20-100		3000ppm/c, large v.
N well	+/- 20%	2k-4k	--	large
1 Contact	+/- 50%	1-3 ohm	---	----

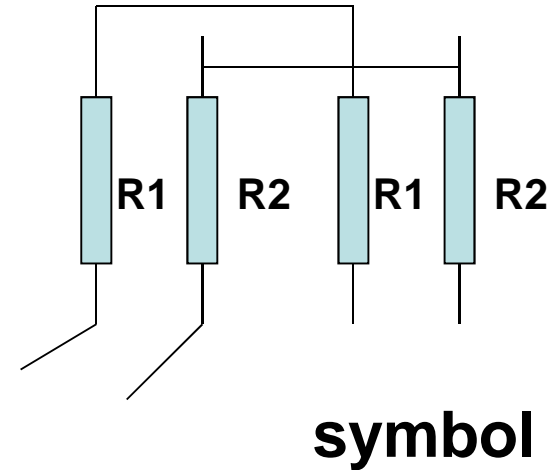
Layout of resistors in silicon



Interdigitized structure :



Good matching



symbol

$$R = R_s \cdot \text{length} / \text{width}$$

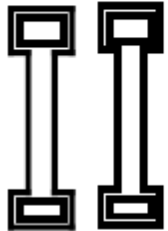
$$R_{\text{total}} = R + 0.5R_s(\text{number of turns}) +$$

+ Contact resistance



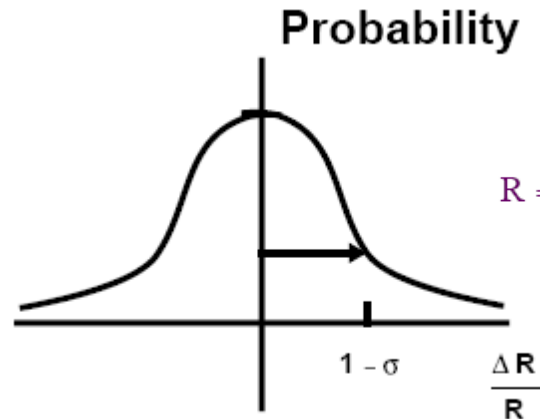


mismatches



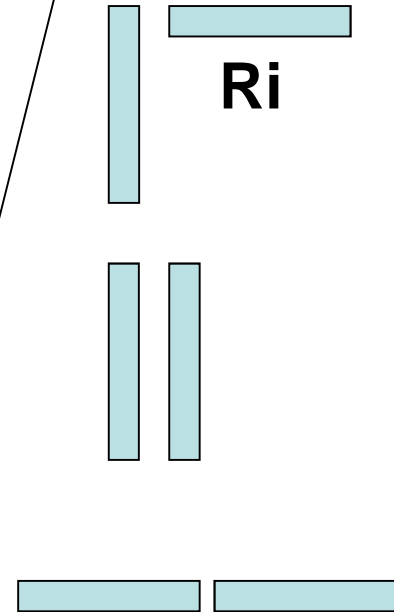
$$R = \frac{R_1 + R_2}{2}$$

$$\Delta R = R_1 - R_2$$



$$R = \frac{L}{W} R_{\square} = \frac{L}{W} \cdot \frac{\bar{\rho}}{x_j}$$

$$\left(\frac{\Delta R}{R}\right)^2 = \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta W}{W}\right)^2 + \left(\frac{\Delta \bar{\rho}}{\bar{\rho}}\right)^2 + \left(\frac{\Delta x_j}{x_j}\right)^2$$



$$\left(\frac{\Delta x_j}{x_j}\right)$$

- Implant dose
- Side diffusivity
- Deposition rate

$$\left(\frac{\Delta L}{L}\right); \left(\frac{\Delta W}{W}\right)$$

- Etching
- Boundary
- Side diffusivity

$$\left(\frac{\Delta \bar{\rho}}{\bar{\rho}}\right)$$

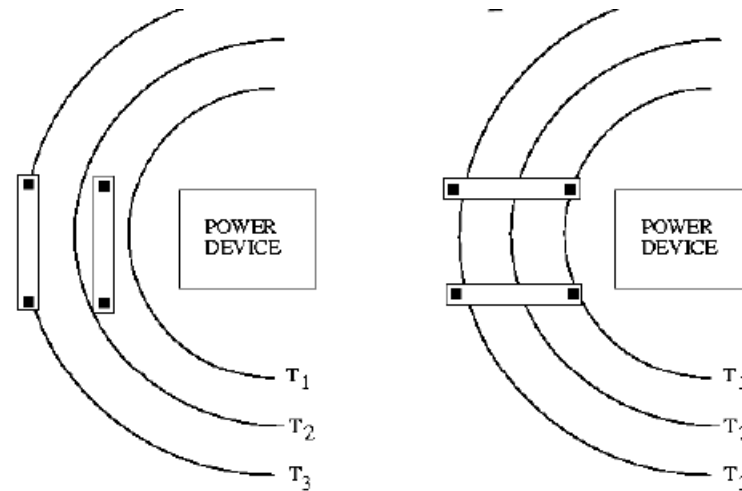
- Polysilicon grain size
- Doping dose
- Crystal defects
- Stress
- Temperature

source: F. Maloberti

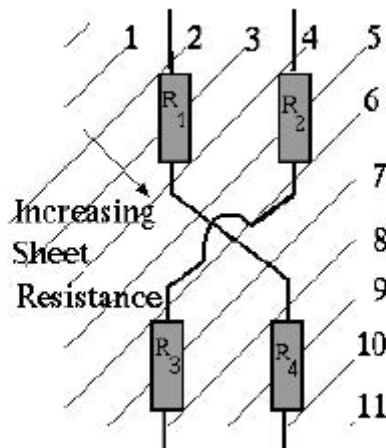
Improve mismatch (accuracy) errors : "good to do"



A- temperature:

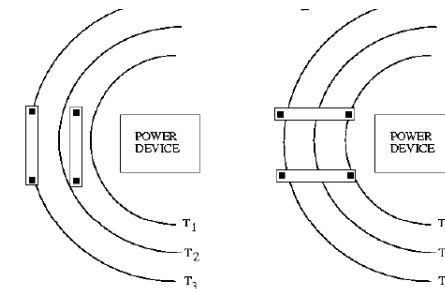


B- sheet resistance



"James C. Daly et al."

source: F. Maloberti



C) STRESS: away from edges..

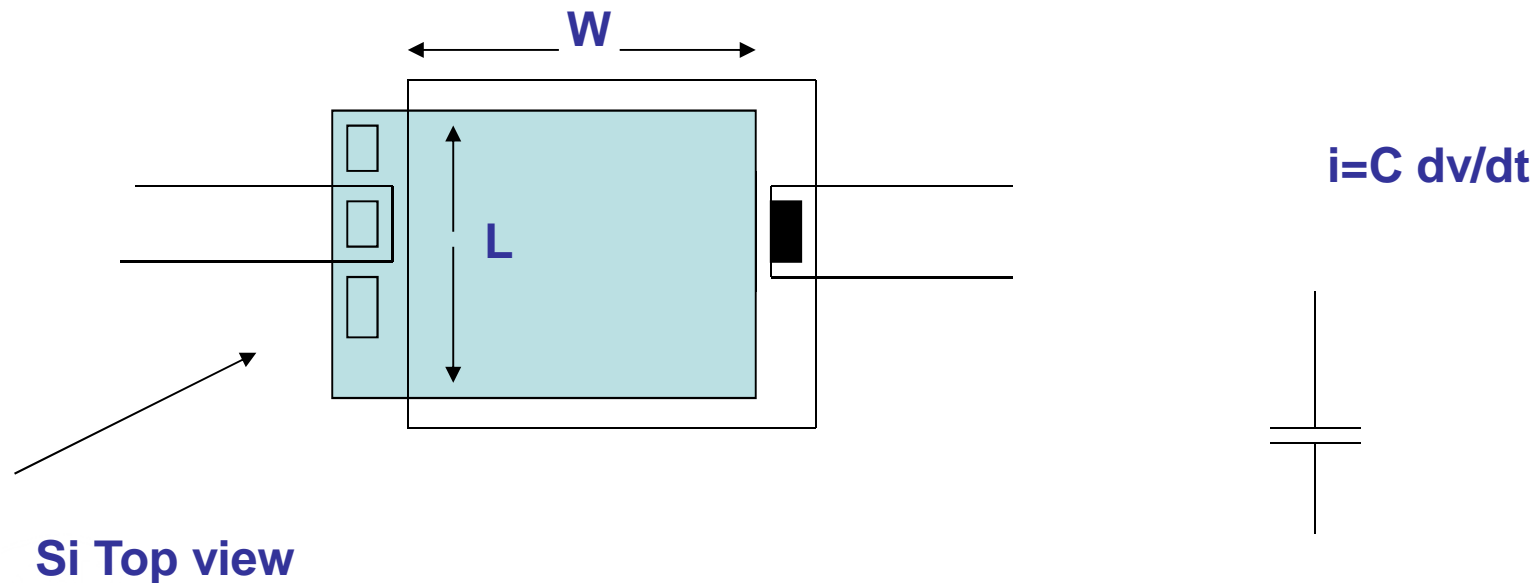
Capacitors



Plated Capacitors

$C = C_a \times \text{Area} = C_a \times W \times L$ (W and L are dimension of plates)
1-4 ff/uu can. 20pF is already a large capacitors for Si application.

“Core Oxide caps”- will be reviewed later lectures.

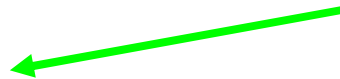


Capacitors



$$C = \frac{\epsilon_0 \epsilon_r}{t_{ox}} WL$$

From here to here..



$$\left(\frac{\Delta C}{C}\right)^2 = \left(\frac{\Delta \epsilon_r}{\epsilon_r}\right)^2 + \left(\frac{\Delta t_{ox}}{t_{ox}}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta W}{W}\right)^2$$

Insulator ~SiO2

Mismatch model: know, but ignore the above use pelgrom rule !
 Given mismatch for certain area everything and 4 time the area means double the matching- next lecture...

$$y = \frac{x_1 \cdot x_2}{x_3}$$

Look @

$$\frac{\partial y}{y} = ?$$

$$\frac{\partial y}{y} \Big|_{x_1} = \frac{\left(\frac{x_1}{x_1}\right) \cdot \partial x_2}{\left(\frac{x_1}{x_1}\right) x_2} = \frac{\partial x_2}{x_2}$$

$$\frac{\partial y}{y} \Big|_{x_2} = \frac{\partial x_2}{x_2}$$

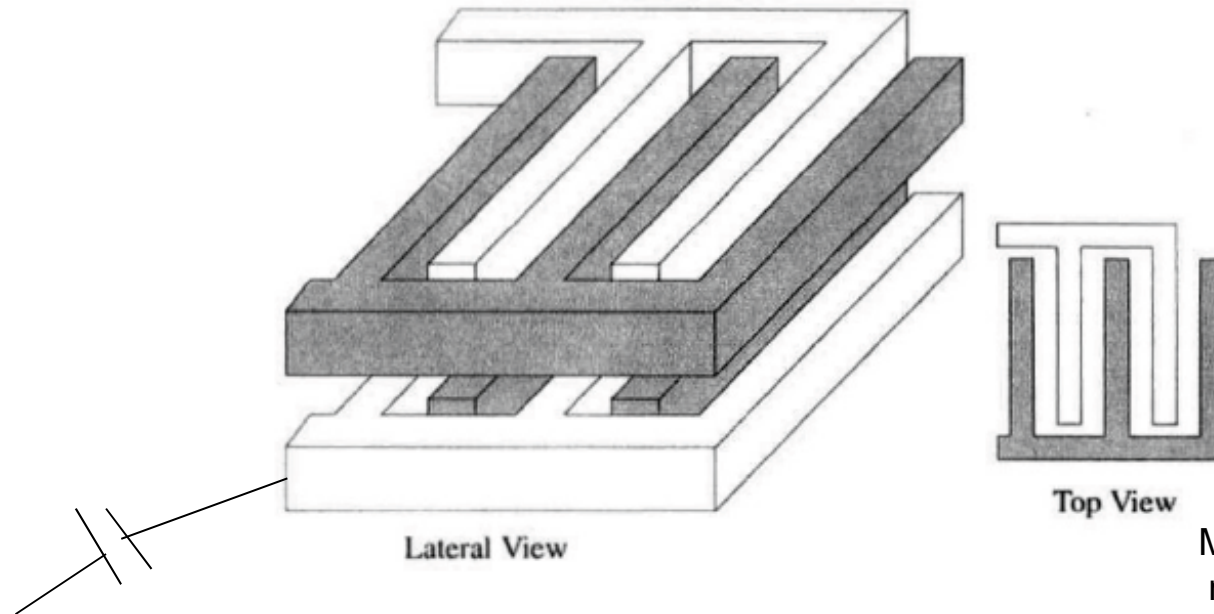
$$\frac{\partial y}{y} \Big|_{x_3} = \frac{\cancel{x_1} \cdot \cancel{x_2} \cdot x_3^{-2} \cdot d x_3 (-1)}{\left(\frac{\cancel{x_1} \cdot \cancel{x_2}}{x_3}\right)^1} = \frac{-1}{x_3} \partial x_3$$

$$\left[\frac{\partial y}{y}\right]^2 = \left(\frac{\partial x_1}{x_1}\right)^2 + \left(\frac{\partial x_2}{x_2}\right)^2 + \left(\frac{\partial x_3}{x_3}\right)^2$$

Metal Capacitors

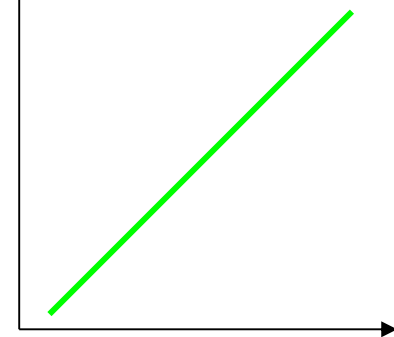


METAL THICKNESS 2000-5000A



Mismatch model: model in area.. – next lecture

delC



$$1/\sqrt{W_{eff} \times L_{eff}}$$

Example : 1-2 e-3 pF/uu

Variations = +/- 20%

Matching dc/c → 10ff ⇒ 0.8%

Temco~0

C to Csub (parasitic) ~ 15:1



END lecture 04

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