Welcome to

**046188 Winter semester 2013**

**Mixed Signal Electronic Circuits**

Instructor: Dr. M. Moyal

**Lecture 02…and 03.**

***Converters Basic Theory and Definitions***

**Definitions/terms- SNR, ENOBs, DNL, INL..**
**And Sampling theory..**

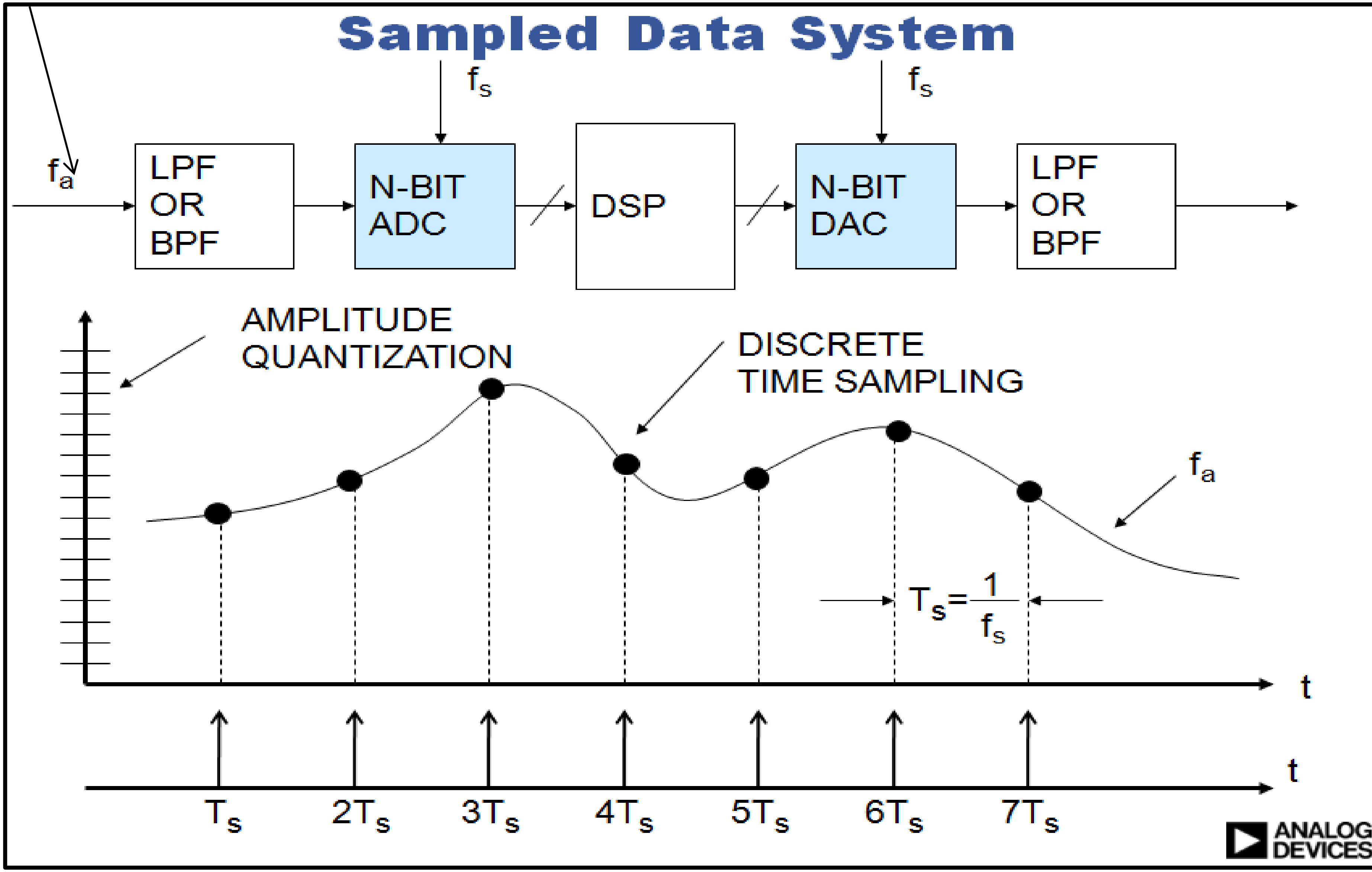
**www.gigalogchip.com**

# Converters Definitions

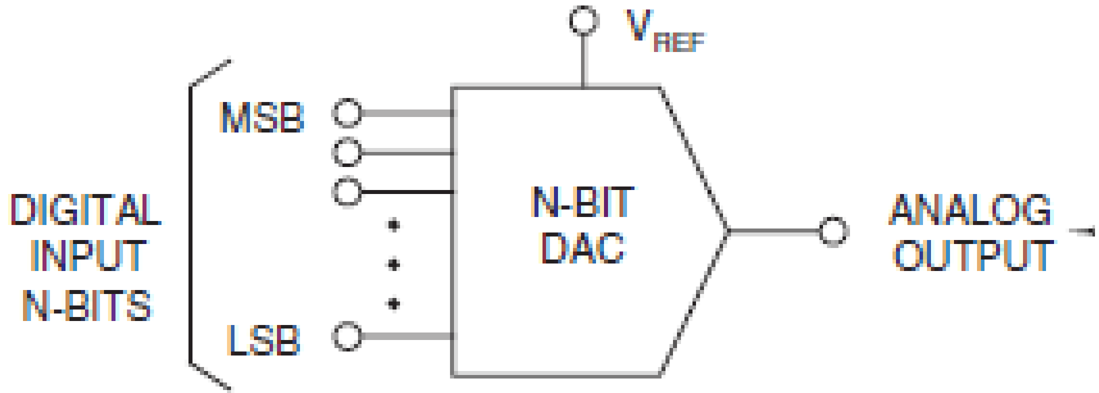
Sample rate and Resolution
Quantization noise (Qn) and Harmonics
QN for Dual tones
SNR- Signal to Noise
DR - Dynamic Range
Distortions: DNL, INL, missing codes
SNRD- signal to noise + Distortions
ENOBs – Effective number of Bits
SFDR- Spurious Free Dynamic Range
FOM
CLOCK Phase Jitter effect on SNR

# Top Building Blocks

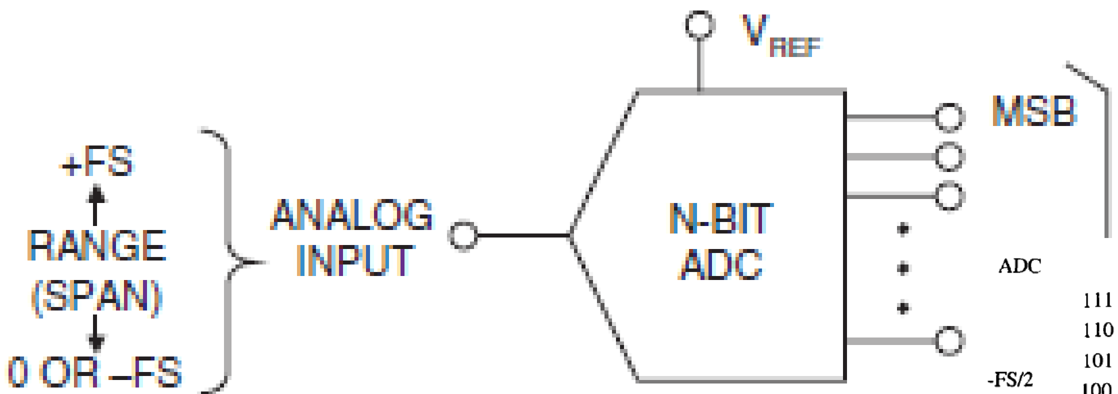
input



# Input/Output..




| +10V FS | BINARY |
|---|---|
| 9.375 | 1 1 1 1 |
| 8.750 | 1 1 1 0 |
| 8.125 | 1 1 0 1 |
| 7.500 | 1 1 0 0 |
| 6.875 | 1 0 1 1 |
| 6.250 | 1 0 1 0 |
| 5.625 | 1 0 0 1 |
| 5.000 | 1 0 0 0 |
| 4.375 | 0 1 1 1 |
| 3.750 | 0 1 1 0 |
| 3.125 | 0 1 0 1 |
| 2.500 | 0 1 0 0 |
| 1.875 | 0 0 1 1 |
| 1.250 | 0 0 1 0 |
| 0.625 | 0 0 0 1 |
| 0.000 | 0 0 0 0 |



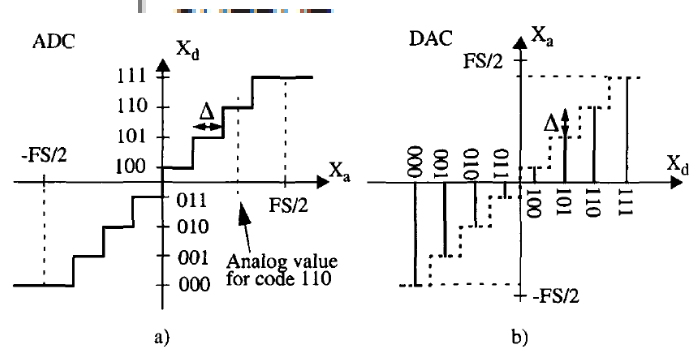




**Figure 1-2** The ADC (a) and DAC (b) transfer functions for N = 3.

# The Sample Rate: output rate: “MS/S”

**Definition: It’s the Rate of digital bits that are coming out**

Mostly it’s the clock rate (non over sampled system).

In Many converters the maximum data frequency is ½ of this.
Depend on signal input maximum BW

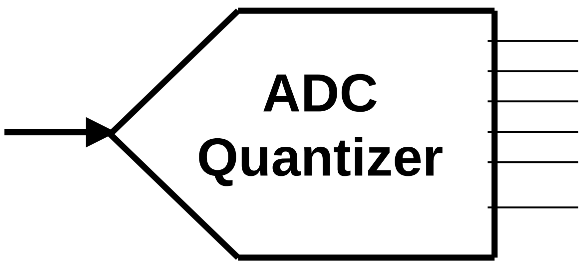
Example: a 10 bit ADC runs at 2MS/S means:

2Ms/s → Output rate is 2mega sample per second, means Sampling clock rate is 2MHz each of the 10 bits rate maximum is 1MHz.

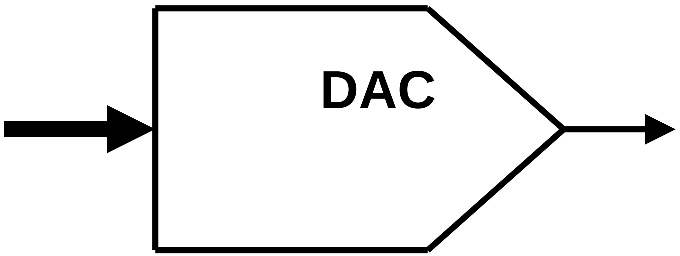
# Resolution

It's the measure of number of digital bit at the output of the converter (ADC).

Its not an indication of the quality
of the converter (bits may or may not move).




Number of
digital bits
or levels



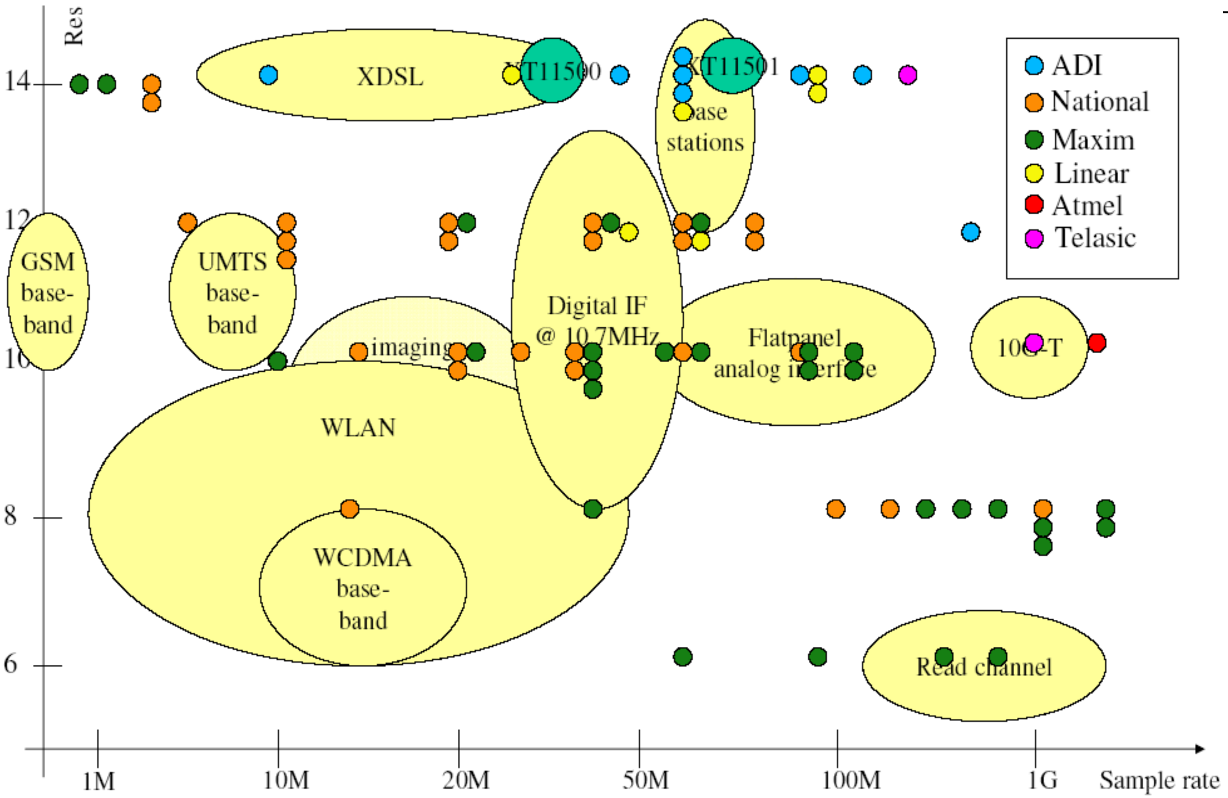

**The number of bits of the digital code is finite, namely n.**

For n bit we have

$2^n$ Possible levels

$2^n - 1$ Possible steps

# Application: data rate and resolution




**Resolution**

**Rate**

**SOME Mixes Signal APLICATIONS**

| | |
|---|---|
| **Wireless LAN** | **1-100MS/s, 6-11b** |
| **Magnetic storage** | **0.2 – 1GS/s , 6-8b** |
| **xDSL** | **1Ms/s – 100MS/S 11-14b ( 30 MHz adc)** |
| **Ultrasound** | **40MS/S 8-12b** |
| **AKG** | **~ Ks/s 18-22bit** |
| **Digital TV** | **20MS/s 8-10b (base band)** |
| **Handy- GSM** | **400MS/s 12b ( base band)** |
| **CATV decoder** | **10-20 MS/s 8-10b (modem ADC)** |
| **HDTV** | **50-100 MS/s, 10b** |
| **1-10GbaseT** | **130MS/s-840MS/s 7b-9b** |

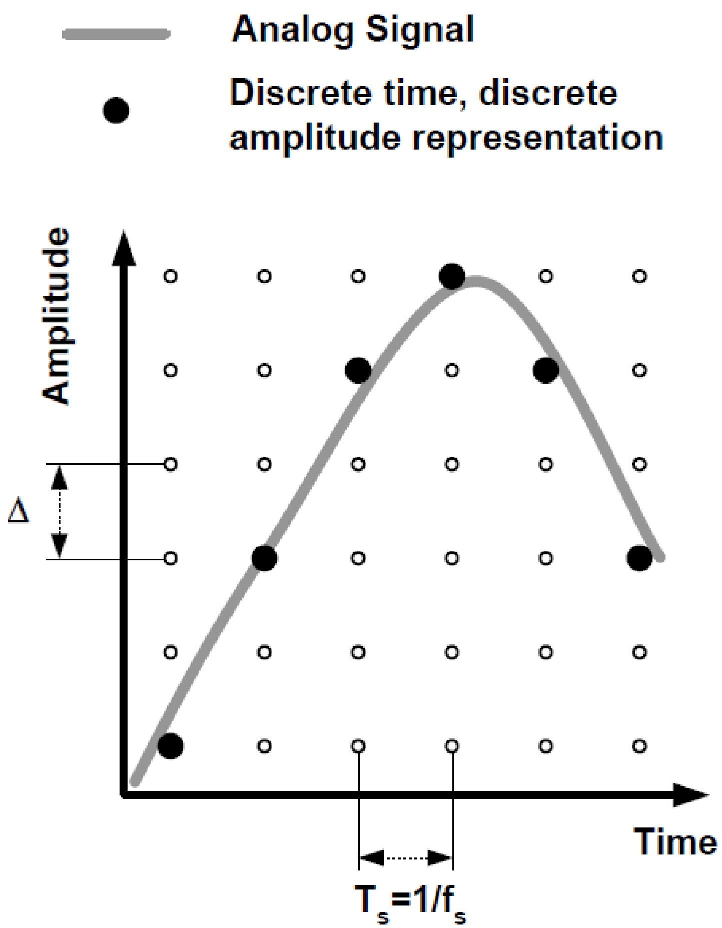
**Videos, Audios…etc.. etc..**

# QUANTIZATION NOISE

The number of bits of the digital code is finite, namely n. For n bit we have $2^n$ possible codes each code represent a given ***quantization level***.

The error due to the quantization is called the ***quantization error*** and ranges between + and – half quantization level (LSB).

This error is one more measure of the ADC quality

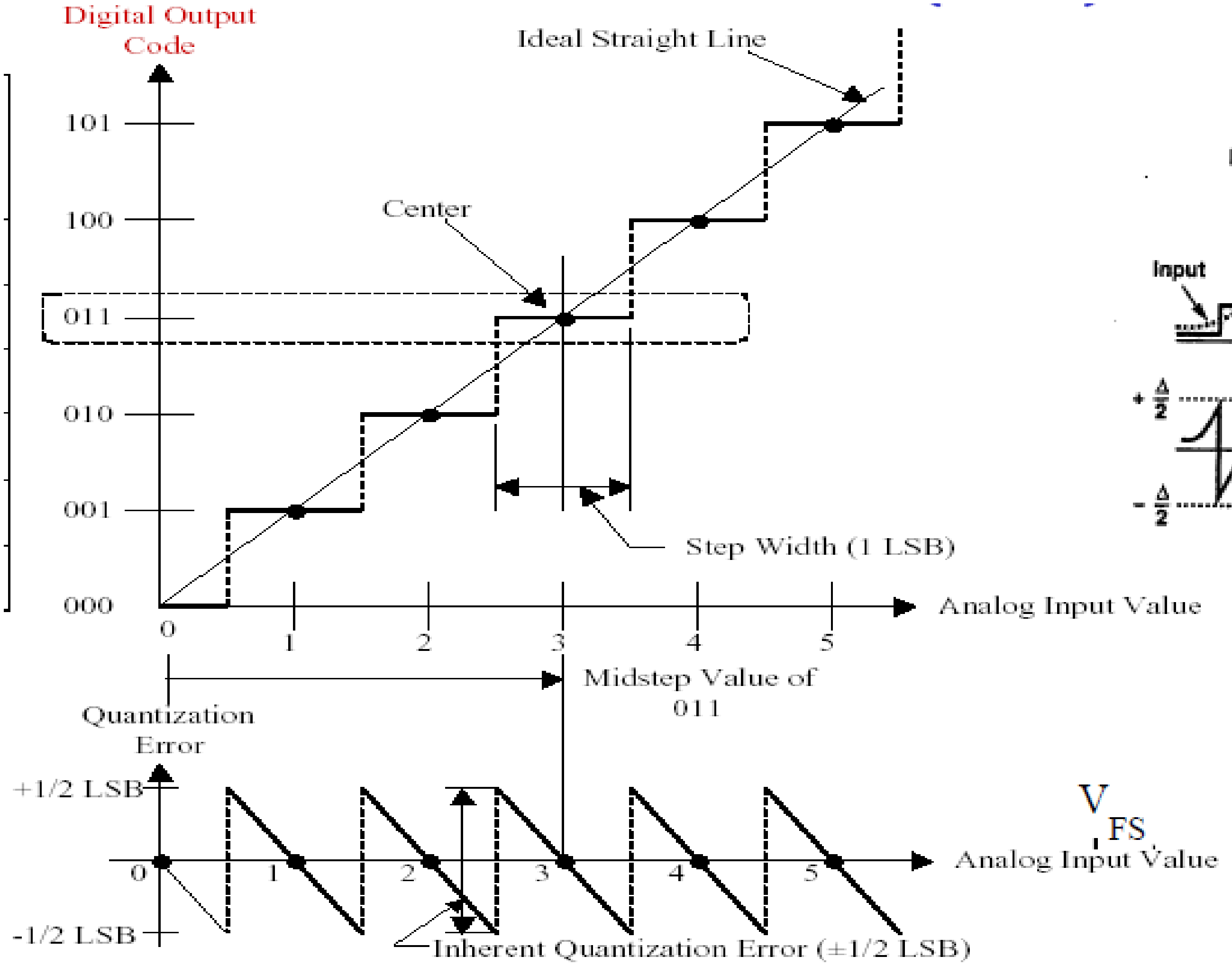



$$\textbf{\textit{possible codes}} = 2^n$$

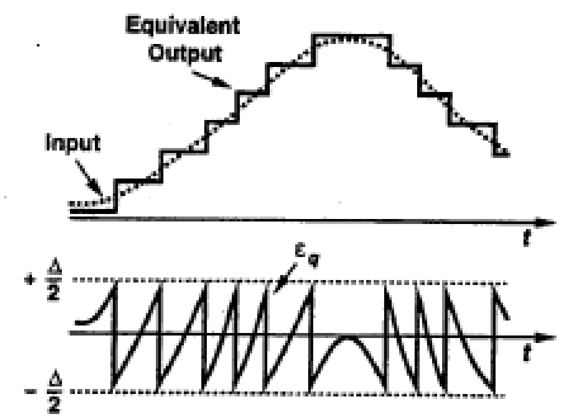
- ***Digital bits are integers: 9, 10, 16 etc..***
- ***Therefore can't represent the***
- ***input signal perfectly: error***

Quantization error cant be higher then the resolution, vice versa is possible

# QUANTIZATION NOISE




sine wave




***Input minus output after gain and offset errors are nulled***

# QUANTIZATION NOISE CALCULATION

*Assuming the input signal has **uniform density function** over each code bin*
*Than quantization noise is well approximated by uniform distribution*
*and white spectrum*

MEAN-SQUARE ERROR = $\overline{e^2(t)} = \frac{q}{s}\int_{-q/2s}^{q/2s}(st)^2\,dt = \frac{q^2}{12}$

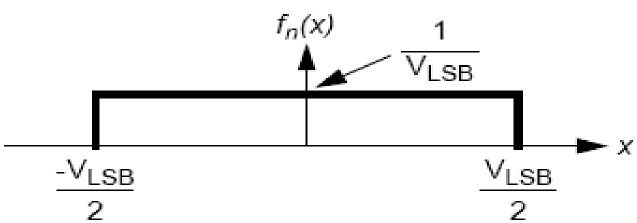
ROOT-MEAN-SQUARE ERROR = $e_{rms} = \sqrt{\overline{e^2(t)}} = \frac{q}{\sqrt{12}}$

## Quantization Noise Power

— deterministic approach (assume input is a ramp)

— stochastic approach (assume rapidly varying input)

probability density function




$\int_{-\infty}^{\infty} f_n(x)\,dx = 1$

un
de
tin

$$Nq = Vlsb / \sqrt{12}$$

***approximately 1/3 of an LSB !***

$$V_{n(rms)} = \left[\int_{-\infty}^{\infty} x^2 f_n(x)\,dx\right]^{1/2} = \left[\frac{1}{V_{LSB}}\int_{-\frac{V_{LSB}}{2}}^{\frac{V_{LSB}}{2}} x^2\,dx\right]^{1/2} = \frac{V_{LSB}}{\sqrt{12}}$$

- **this noise power is spread between $-f_S/2$ and $f_S/2$**

# QUANTIZATION NOISE- Cont. in term of full scale

*Full scale voltage is the parameter we're interested in. To maximize or distribute all the available codes we split the full scale (Vpk) to all the possible codes.*

$$Vfs = Vlsb \bullet 2^{n} - 1$$

Representing the quantization error by an additive noise is a critical approximation: it models a non-linear effect with a linear process.

***Substitute into the quantization noise Eq.***

$$Nq = Vlsb / \sqrt{12}$$

**Quantization noise**

$$n_Q^2 = \frac{VFS^2}{12 \cdot 2^{2 \cdot Nbit}}$$

***A sine wave for example at the end point ( slowly moving input) may not be uniform enough over the code bin.***

Sample rate not repeated "close" to signal frequency or Nq. will not have enough information..

# SNR

*Definition SNR*

*In telecommunication the output quality is measured in term of Signal to Noise Ratio (SNR)*

*Definition: SNR is defined as the ratio of output signal So power to the base band noise power at the output No. Including quantization, Harmonics (sometime not), and all flicker thermal jitter noises.*

$$SNR = 20\log(Vin(rms) / Vq(rms))$$

$$Y(kTs) = X(kTs - kTd) + No(kTs)$$

$$No = E\{No(kTs)\}^2$$

$$So = E\{X(kTs)\}^2$$

$$SNR \equiv \frac{S_0}{N_o} = \frac{E\{X^2(kT_s)\}}{E\{n_o^2(kT_s)\}}$$

$$SNR \equiv (So / No)$$

$$SNR(dB) \equiv 10 \bullet \log(So / No)$$

*Source: K.S.Shanmugam*

# Theoretical Quantization Noise Ideal N-Bit Converter

***let : q=Vlsb***

$$SNR = 20 \cdot \log_{10}\left[\frac{Full\_Scale\_Sinewave\_(rms)}{Quantization\_Noise\_(rms)}\right]$$

Signal power = mean square root

$$v_{in}^2 = \frac{1}{2\pi}\int_0^{2\pi} A^2 \sin(\omega t)^2 dt = \frac{A^2}{2}$$

out

$$v(t) = V_0 \cdot \sin(\omega \cdot t) = \left[\frac{q \cdot 2^N}{2}\right] \cdot \sin(\omega \cdot t)$$

$$v(rms) = \frac{V_0}{\sqrt{2}} = \left[\frac{q \cdot 2^N}{2 \cdot \sqrt{2}}\right]$$

$$SNR = 20 \cdot \log_{10}\left[\frac{q \cdot 2^N}{2 \cdot \sqrt{2}} \cdot \frac{\sqrt{12}}{q}\right] = N \cdot (20 \cdot \log_{10} 2) + 20 \cdot \log_{10}\sqrt{\frac{3}{2}}$$

MEAN-SQUARE ERROR = $\overline{e^2(t)} = \frac{q}{s}\int_{-q/2s}^{q/2s} (st)^2 dt = \frac{q^2}{12}$

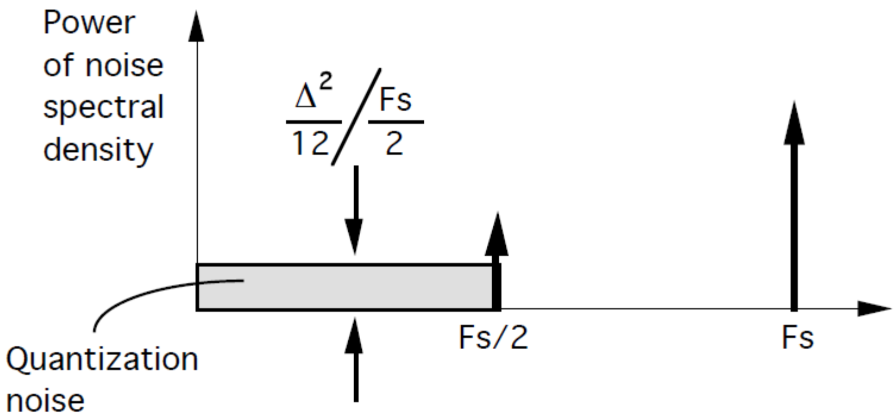
ROOT-MEAN-SQUARE ERROR = $e_{rms} = \sqrt{\overline{e^2(t)}} = \frac{q}{\sqrt{12}}$

$$SNR(dB) = 6.02N + 1.76$$

$$ENOB = \frac{SNR(dB) - 1.76}{6.02}$$

"not fully true"

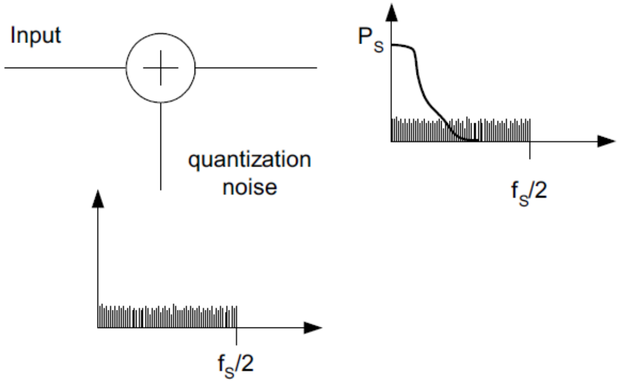
- It is assumed that the quantization noise exhibits a white spectrum




A. Baschhirotto, LVA design 2011.

- The power-of-noise spectral density is:

$$n_q^2(f) = \frac{\frac{\Delta^2}{12}}{\frac{F_s}{2}}$$




Sample rate not repeated close to signal frequency or Nq.  will not have enough information..

- ***Key: The noise is spread:   to +/- fs/2   ( Nyquist interval)***
- *or 0-fs/2      (representation)*

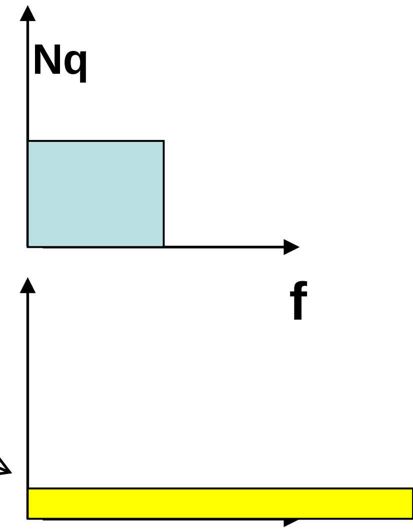
# QUANTIZATION NOISE DENSITY: EXAMPLE

*Key: How far does it spread and how does it depend on frequency?*

*The quantization noise spreads to the half of the clock frequency. (+ / - fs/2 same as 0-fs/2 )*
*That is to say we can define quantization noise per root hertz. And now get the*
*Total noise for a fixed BW that we operate in. ( a must for non nyquist converters)*

**EXAMPLE1** :
*a) If LSB is 1 mV and we sample at 2 MHz: 288uV is spread over 1 MHz. which means*
*0.288uV/sqrtHz ( 288u-6/ sqrtHz{(1e6)}*

*b) If we sample at 16 MHz the quantization density is : 0.101 uV/sqrHz*
*(divide by sqrt(8). 0.288/2.82.*



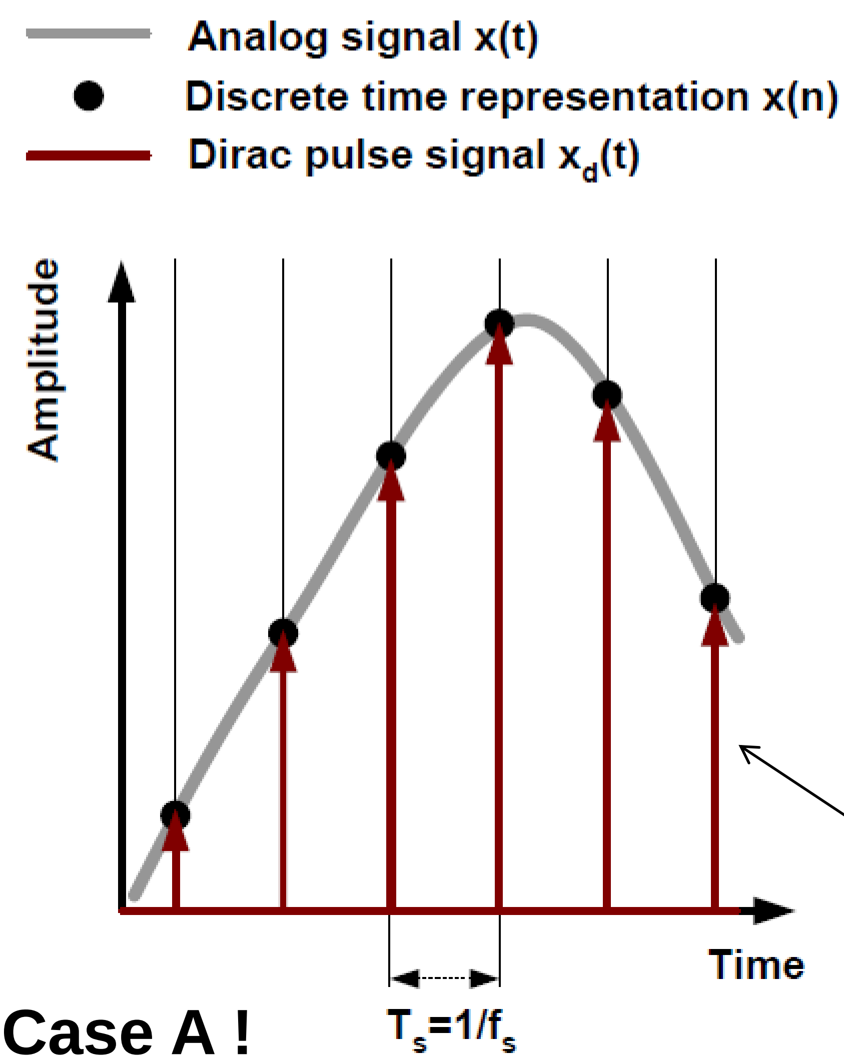

***Conclusion***
*Good to increase the sampling clock we profit:*
*10 log (fs/ fsignal BW) = 3dB/octave !*

*Example2 ( the dB )*
*10 bit adc with max input BW=1MHz and 2MHz sampler quantization noise is: ~60 dB*
*10 bit adc with 1MHz BW and 16MHz sampler quantization noise is: ~69 dB*

**10 * log8**

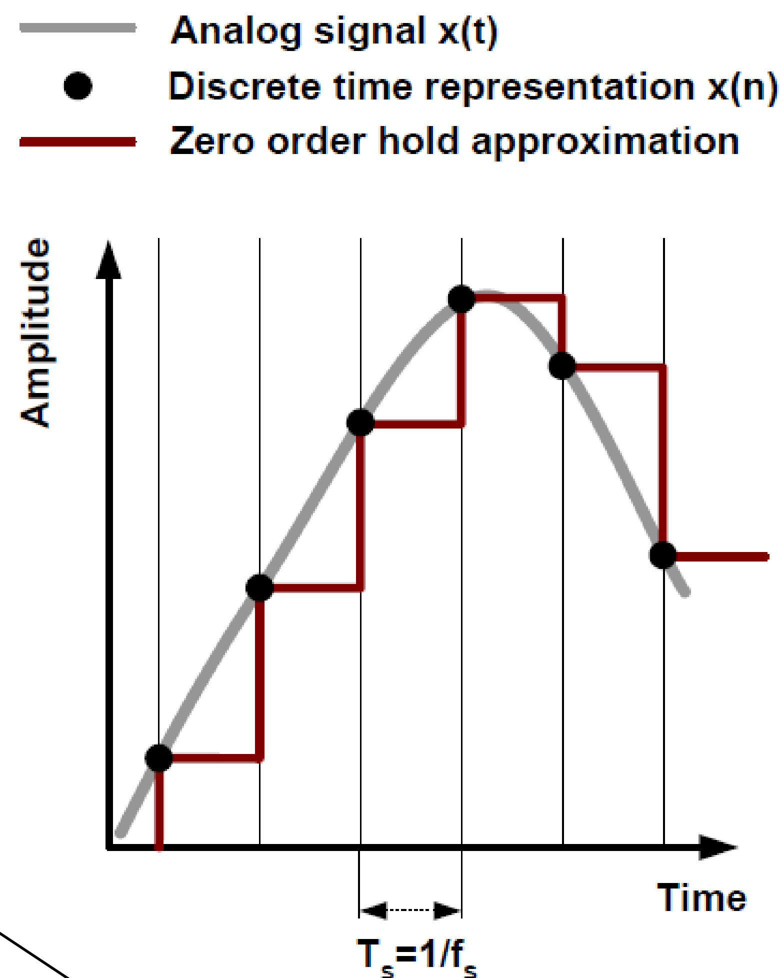
## SAMPLING PROCESS OVERVIEW

# Track and hold outputs– “perfect” track



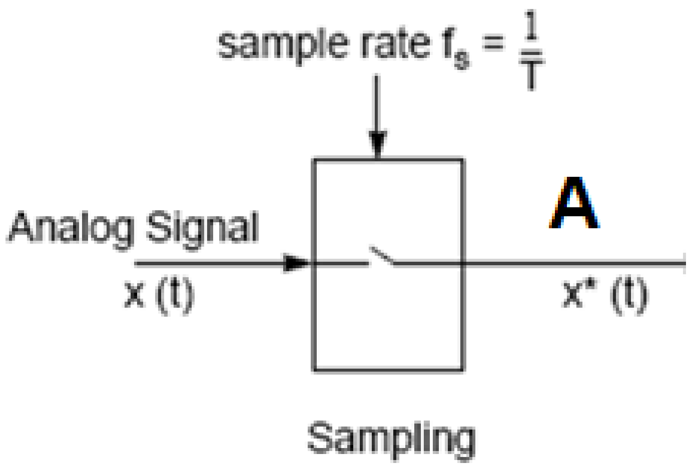

**Case A !**

$$x_d(t) = x(t)\cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

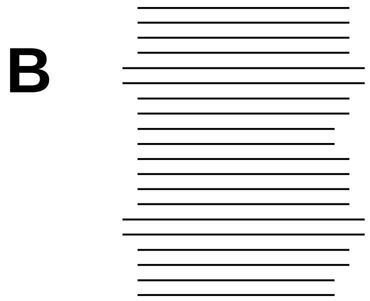



**Case B !**

Hard to design a “delta” function




**Here fixed levels**




$$x^*(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT)$$

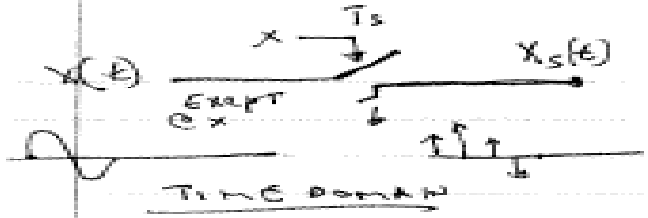
*where:*

$$\delta(t) = 1, \quad t = 0,$$
$$\quad 0, \quad elsewhere$$

**Math. model**

**What is the difference at point A and B ? …( Nq)**

# Case A !




$= x(t)$ ⊗ $\delta(t - nT_s)$

SAMPLING WITH DELTA FUNCTION

$$X_s(t) = x(t) \cdot \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \quad , \quad \int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

also since $\delta(t) = 0$ everywhere except @ $t = 0$.

$$X_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s)$$

$F[\cdot]$ ⇒ defined as Fourier operation.

$$F[X_s(t)] = X_s(f) = X(f) * F\left[\sum_{n=-\infty}^{\infty} \delta(t - nT_s)\right]$$

$$\Downarrow \quad f_s \cdot \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

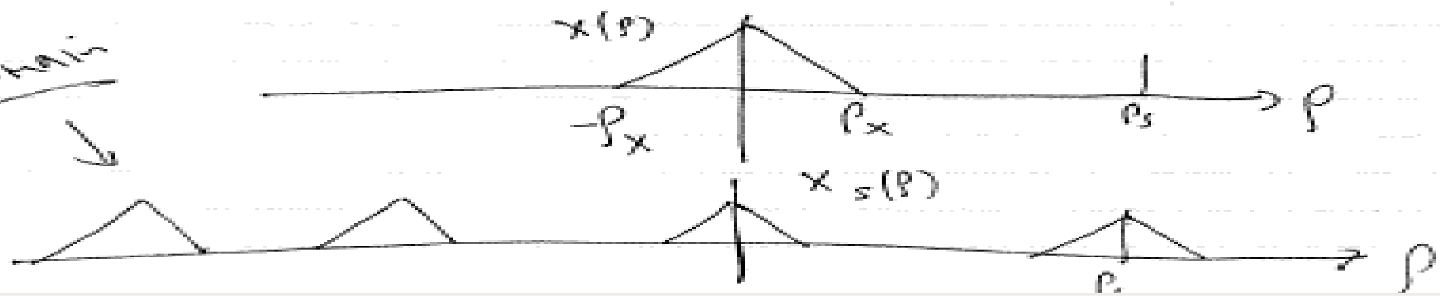
need to prove it.

$$\Rightarrow X_s(f) = f_s \cdot \sum_{n=-\infty}^{\infty} X(f - nf_s) =$$

$$f_s X(f) + f_s \cdot X(f - f_s) + f_s X(f - 2f_s) + f_s X(f - 3f_s) + \ldots$$

$$f_s X(f + f_s) + f_s X(f + 2f_s) + f_s X(f + 3f_s) + \ldots$$

f-domain



Any periodic signal can be constructed from sum of sine waves.

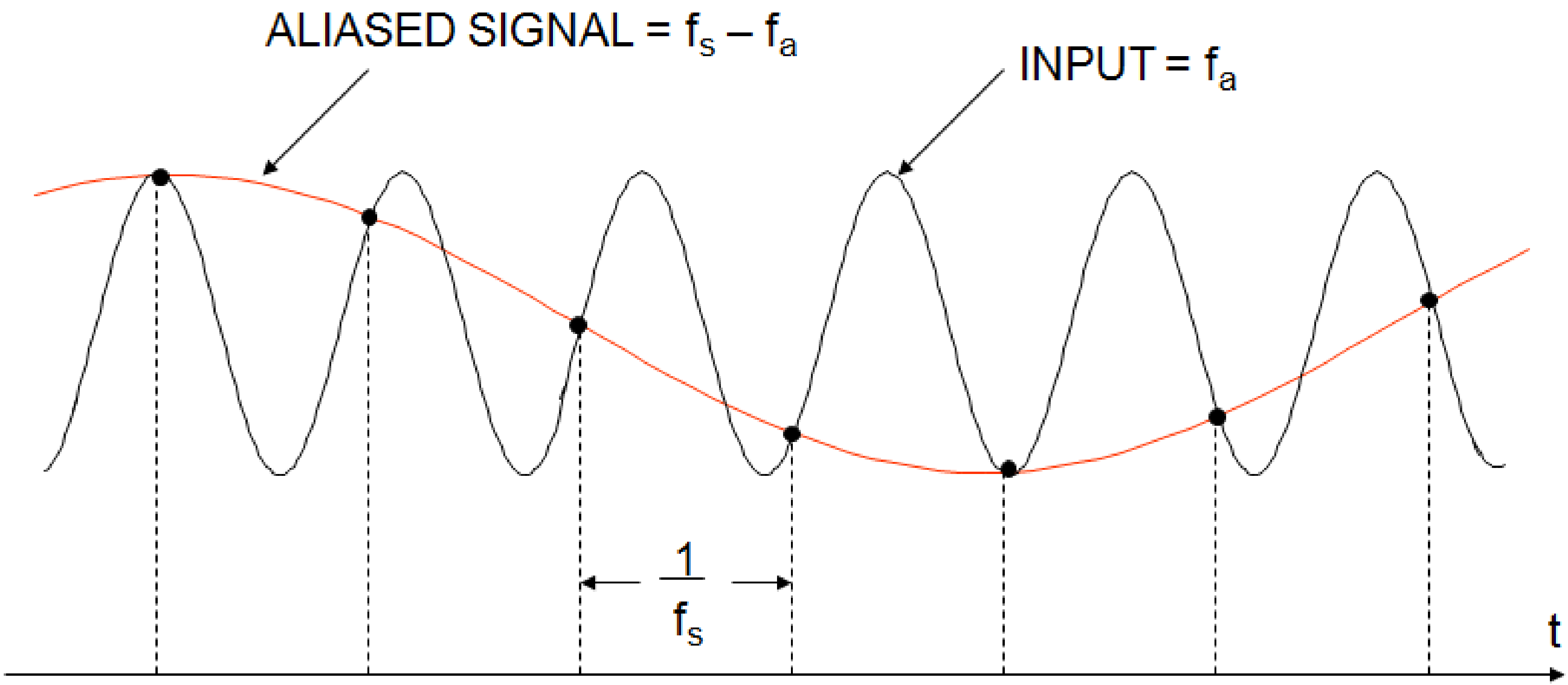
The power (or P.S.D) - Density

is $$S_x = \underbrace{\int_{-\infty}^{\infty} |C_x(nf_0)|^2 \delta(f-nf_0)}_{G_x(f)}\, df = \text{power}$$

also $$\lim \frac{1}{T}\int_{-T/2}^{T/2} |x^T(t)|^2\, dt$$

**Key: The power of X(t) is the same in f domain= sum of the coefficient**

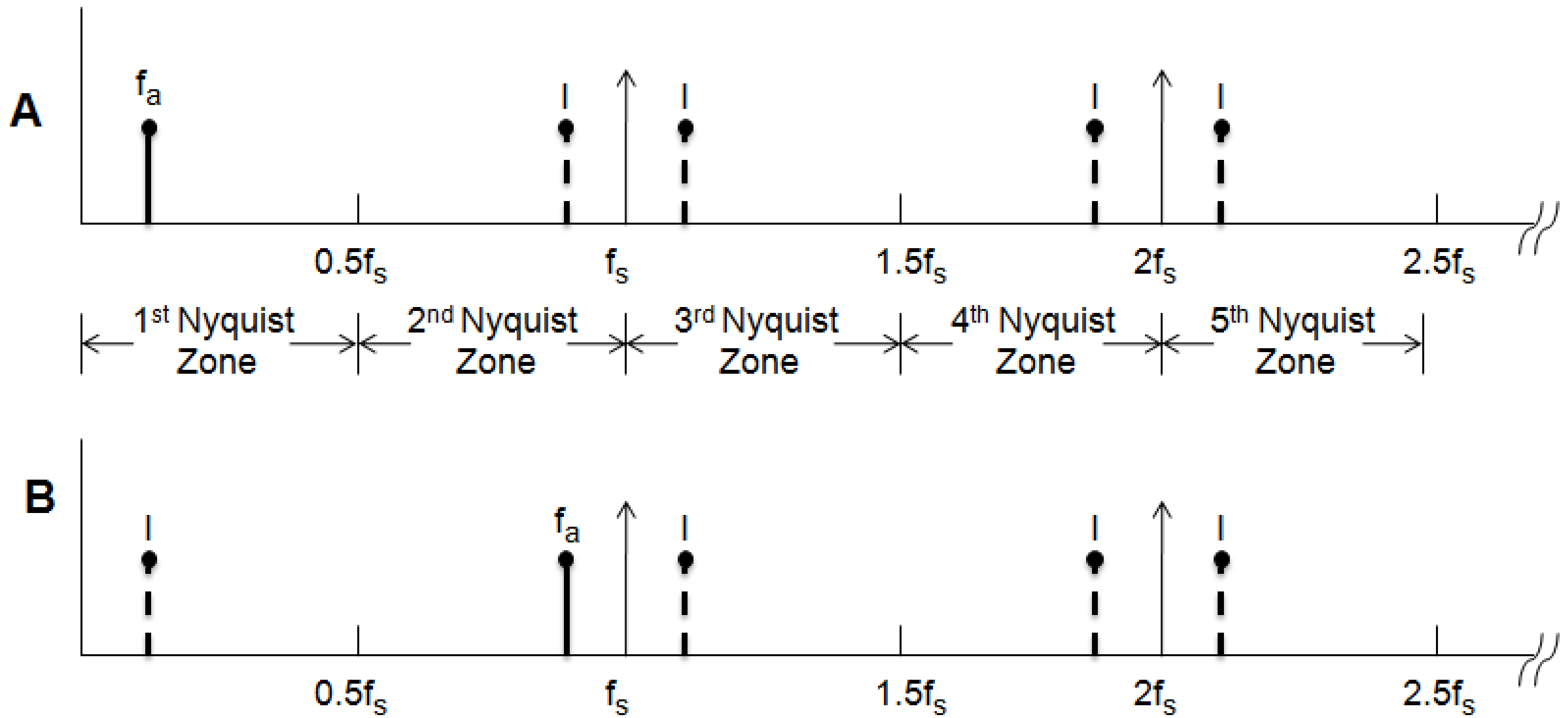
# Aliasing in the Time Domain




NOTE: $f_a$ IS SLIGHTLY LESS THAN $f_s$

# Example

## Aliasing in the Frequency Domain




**fa is the input signal sampled at fs**

# Sampling: the Shannon Theorem

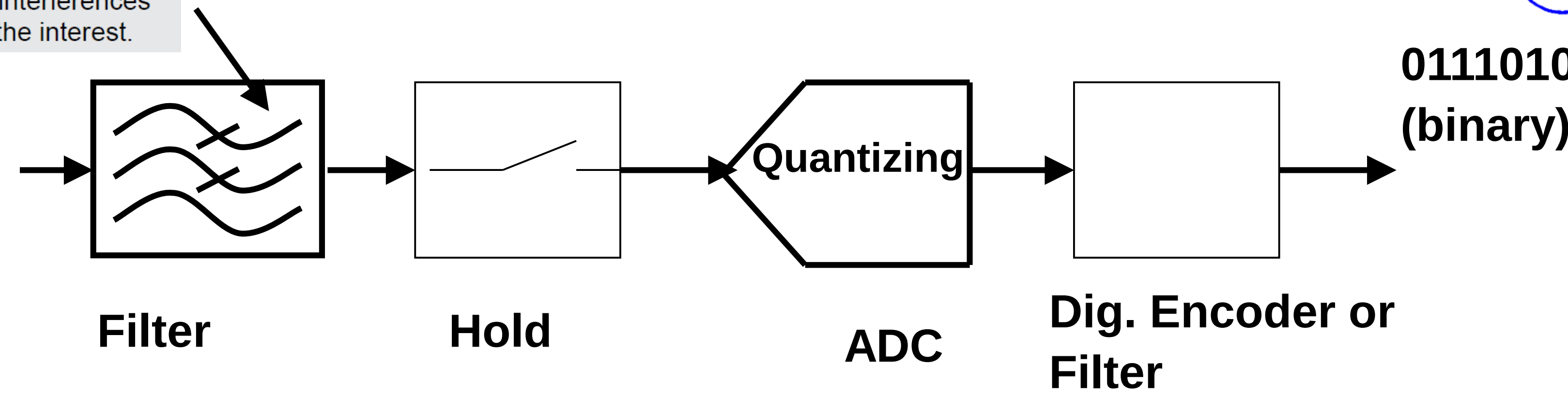
• The Shannon Theorem says: "If a signal x(t) has a **Limited Bandwidth (-BW,BW),** it can be univocally determined by its samples x(nT) if the **Sampling Frequency is at least twice the Bandwidth:**
**$f_s = 1/T \geq 2BW$"**

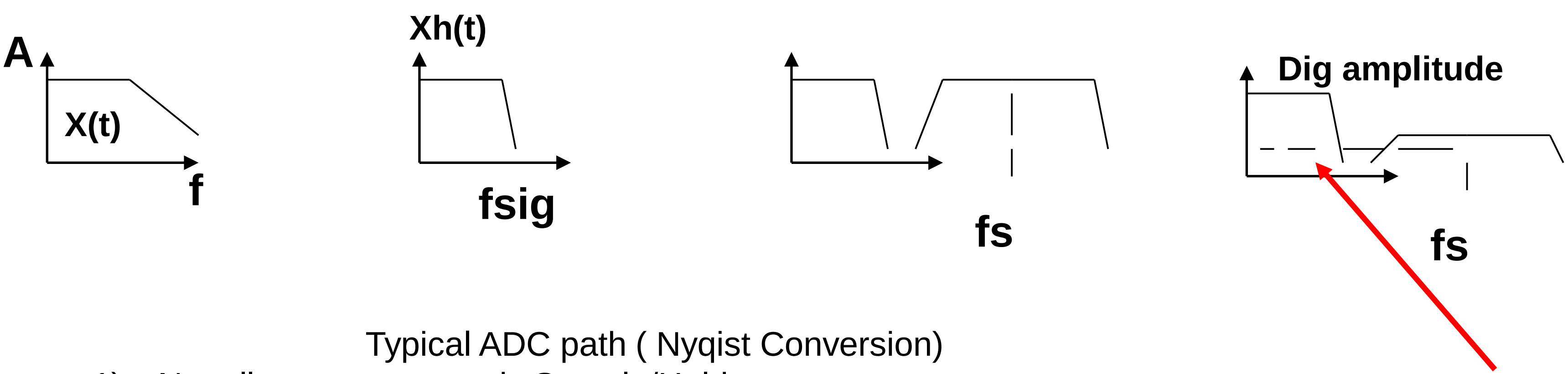
• Note:
1) Limited Bandwidth is a **Necessary but not Sufficient** condition
2) $1/T \geq 2BW$ is only a **Sufficient but not Necessary** condition

Shannon
1949

# Converter Building Blocks

The anti-aliasing filter protects the information content of the signal. Use an anti-aliasing filter in front of every quantizer to reject undesired interferences out of the band of the interest.




Noise: random

systematic, lin

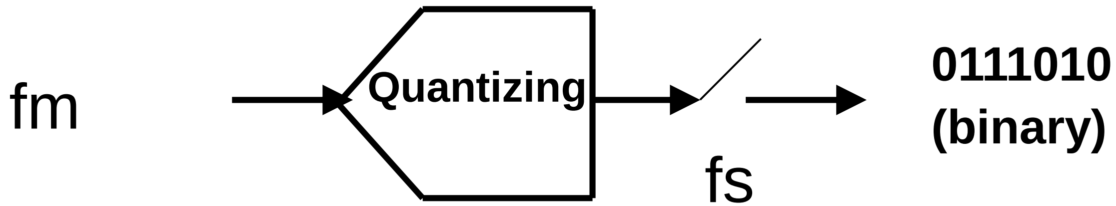
Typical ADC path ( Nyqist Conversion)

1) Not all converters needs Sample/Hold
2) Not all Converters needs LPF, However some also use BPF ( or DC remover)
3) Fsignal coming to the converter is Bounded.
4) ADC output may or may not have reduced folding – but it has noise

**KEY: How each component works, its transfer function, what is the optimum ?**
**first to the definitions ! (lect. 2)**

# CLASS OF CONVERTERS

# Nyquist Converter




Fm=maximum
"Through put" frequency

**ADC**

**fs~2fm**

**Nyquist converter: max speed lowest clock**
**2fm(2xBW) < fs 2fm very close to fs.**

Remember:
S&H not always needed
LPF: Not always needed

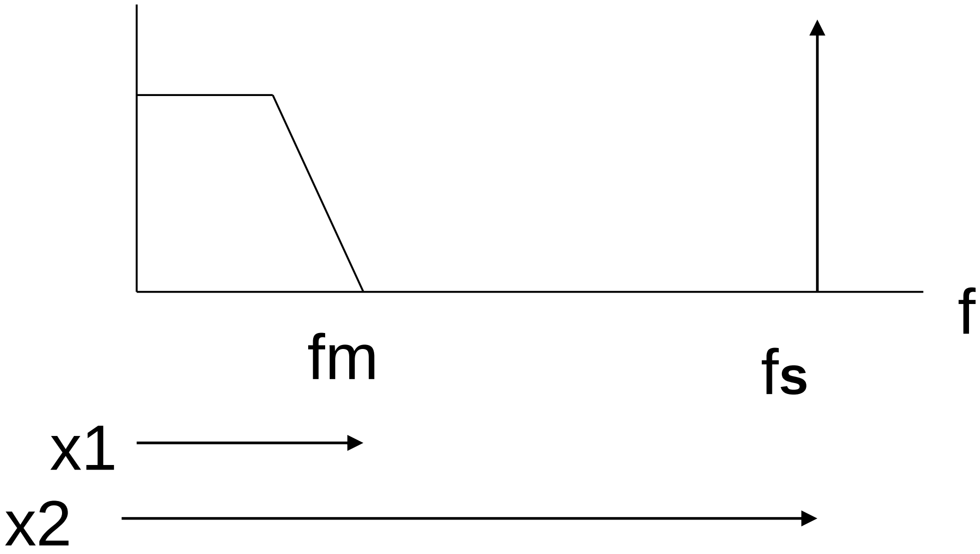
Nyquist
1928

**over sample converter:  max speed lowest clock**
**2fm < fs**

**x2>>2x1**

**fs>>2fm**

**Lecture 7,8,9**

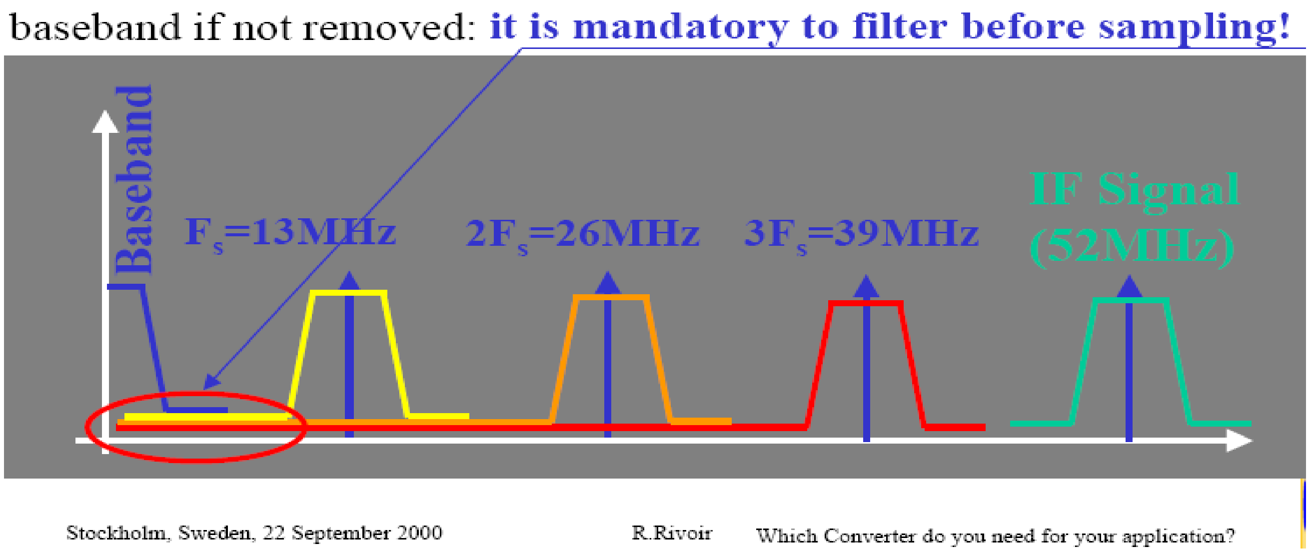



**hint:**
**LPF: may be eliminated or simplified**
**What about T/H ?**

We sample many time over (16x..1024x..)

# Under sampling Converter

**Sample at low clock converter: max speed lowest clock 2fm < fs**



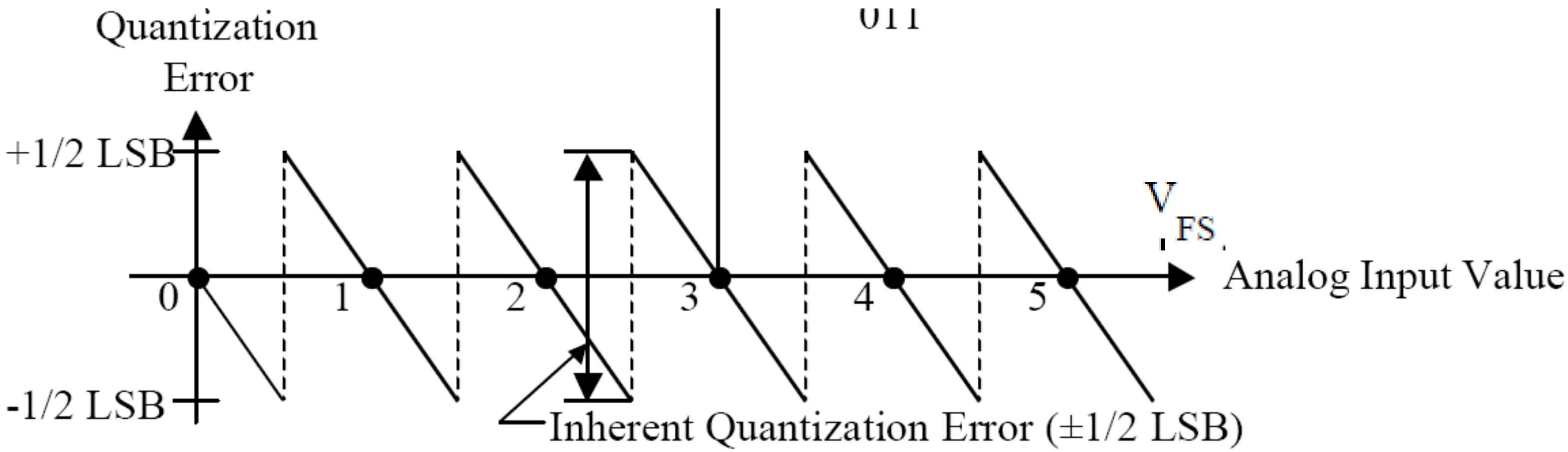

**signals placed at high frequency with band limitation can be reproduced with low rate clock.**
**Without contradiction to sampling theory.**
**The original signal spectrum folds in the base band**

BW of signal is the limitation only, not its location (BPF)

**But: Design must take care of the fastest signal ( slewing, bw etc..)**

***Can quantization produce non linear output signal? – Yes.***

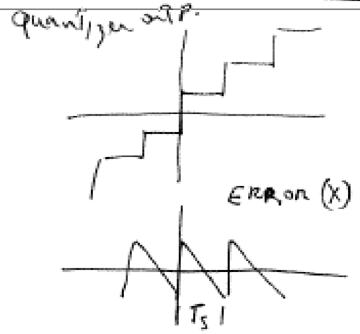
*We measure its  Harmonics ? Non linearity's ?*




Elements of Transfer Diagram for an Ideal Linear ADC

# QUANTIZATION NOISE HARMONIC DERIVATIONS

Quantization Error Spectra




$Y_{in} = \cos \omega_m t$

USE F. Trasform.

$$\text{Error}(x) = \frac{T_s}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin[n \omega_x Y_{in}(x)]$$

$$\text{Error}(t) = \frac{T_s}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(n \omega_x \cos \omega_m t)$$

$$= \frac{T_s}{\pi} \sum_{k=0}^{\infty} a_{2k+1} \cos[(2k+1) \omega_m t]$$

$$\Rightarrow a_{2k+1} = \frac{2T_s}{\pi} (-1)^k \sum_{n=1}^{\infty} \frac{J_{2k+1}(n\omega_x)}{n}$$

Source UCLA

$a_{2k+1} = \frac{2T_s}{\pi} (-1)^k \dots$ = HARMONIC Level

$J_{2k+1} \dots$ = Bessel function

RESULTS

$$a_3 = 2^{-n^{3/2}}$$

***Full derivation in page 12 in the book, only 3rd H. due to trangle error shape.***

***Signal: is "frequency mpdulated by the error"***

***Conclusion***

***Example***
***10 bit produces 15 bit harmonic sat, -90dB from full scale.***

***16 bit converter will have ~-24x6.02dB third order distortions***

*Intermediation distortions (IMD):*

*When we apply to a converter two signals f1 and f2 close in frequency. The amount of distortions due to the converter digitizing the signals is specified as :*

***Full derivation in page 18-19 in the book, only 3rd H. due to triangle error shape.***

***Remember the results.***

$$\text{IMD} = 20\text{Log}_{(10)} \frac{\text{RMS sum of distortion terms}}{\text{Input (Volts, RMS)}}$$

where the distortion terms are given by

2nd-order terms: - f1 + f2, f1 - f2
3rd-order terms: - 2f1 + f2, 2f1 - f2, f1 + 2f2, f1 - 2f2

**Called : Cross Modulation…(the IM3)**

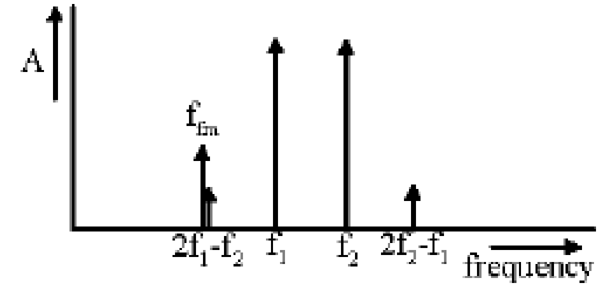
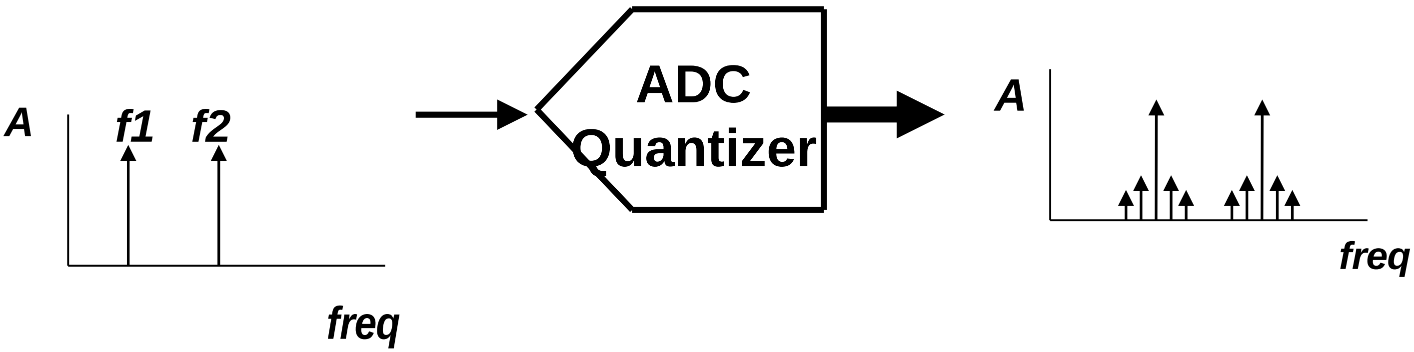
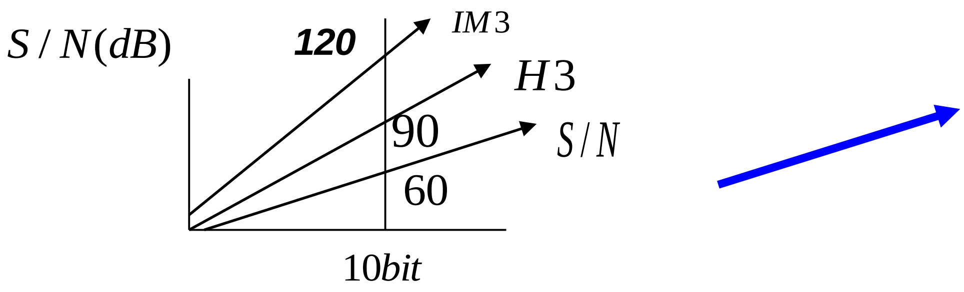



Figure 2: Third-order intermodulation effect

# QUANTIZATION NOISE HARMONIC MORE THAN 1 TONE




A




$2^{-n}$ $2^{-1.5n}$ $2^{-2n}$

*Example*
*10 bit ADC produces "20 bit IM harmonic" IM3 at -120dB from full scale.*
*"Almost" ok to ignore.. When using over 10bit converters..*

# Summary: SNR due to quantization cont.

$$SNR_{sine}\big|_{dB} = (6.02 \cdot n + 1.78)\ a$$

$$SNR_{triang}\big|_{dB} = (6.02 \cdot n\ )dB$$

| number of bits n | S/N Accurate dB | S/N n 6.02 + 1.76 dB |
|---|---|---|
| 1 | 6.31 | 7.78 |
| 2 | 13.30 | 13.80 |
| 3 | 19.52 | 19.82 |
| 4 | 25.59 | 25.84 |
| 5 | 31.65 | 31.86 |
| 6 | 37.70 | 37.88 |
| 7 | 43.76 | 43.90 |
| 8 | 49.82 | 49.92 |
| 9 | 55.87 | 55.94 |
| 10 | 61.93 | 61.96 |

Table 1.1: S/N as a function of the number of bits n

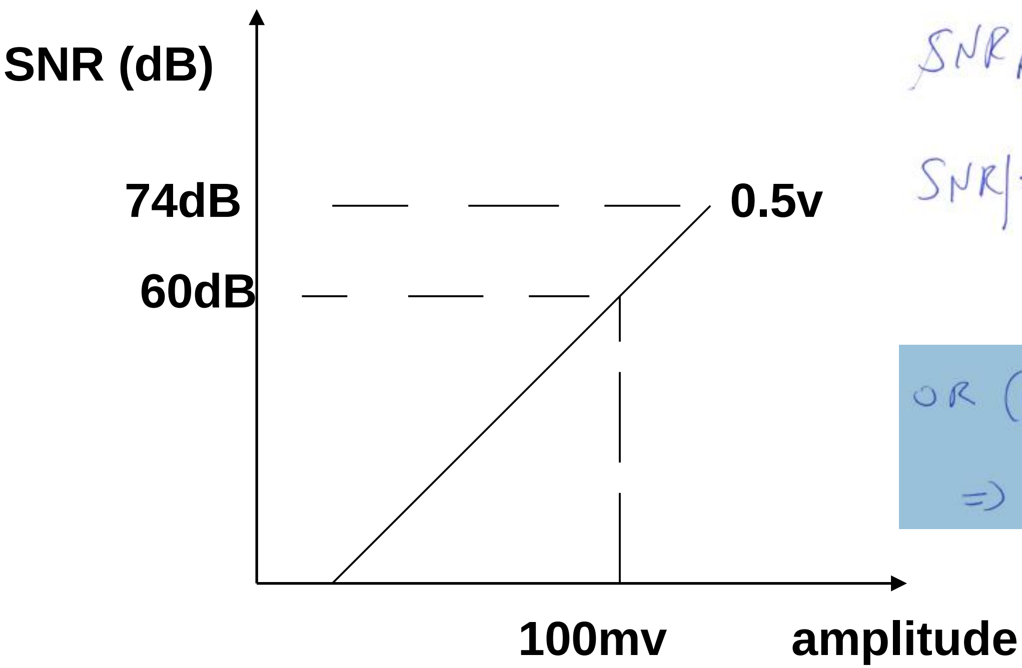
The signal-to-noise of an n-bit converter is accurately modeled with:

$$S/N(n) = \frac{A_1}{A_{quantization}} = \frac{2^{n-1} + \sum_{m=1}^{\infty} \frac{2}{m\pi} J_1(2m\pi 2^{n-1})}{\sqrt{\sum_{q=1}^{\infty} \left(\sum_{m=1}^{\infty} \frac{2}{m\pi} J_{2q+1}(2m\pi 2^{n-1})\right)^2}} \qquad (1.45)$$

will prove n=1 later in the course

***Remember:***
***But it is not exact for 1-4 bit there is some deviation ( 1bit: 6.31dB instead of 7.78 dB)***
***Above 4 bits the error is in the second digit point of the SNR***

*Example 100mV sine wave is applied to an Ideal 12b converter which has its maximum range at 1V. Find the SNR of the digitized output, plot it*

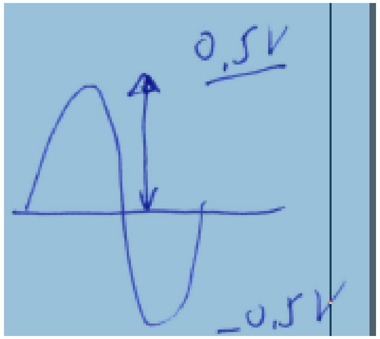



$SNR_{max} = 6.02n + 1.76 = 74\,dB$

$SNR = 10\log\frac{\left(\frac{0.1^2}{2}\right)\cdot 12}{1/2^{12\cdot 2}} = 60\,dB \leftarrow 10\log\frac{A^2/2}{\delta^2/12}$

OR (another way 100mV is 14dB below 0.5V (20 lg 5)

$\Rightarrow 74 - 14 = 60\,dB$




$LSB = \frac{1}{2^{12}}$

*Some input are not sine waves but a complex waveform QAM which have much higher signal peak to RMS value.*
*In that case SNRpk represent the peak value to the RMS noise..*

# DISTORTIONS IN CONVERTERS

# (beside Quantization Noise)

### 1.6.4 Total Harmonic Distortion (THD)

The total harmonic distortion (THD) is the ratio of the total harmonic distortion power and the power of the fundamental in a certain frequency band, i.e.

$$THD = 10 \cdot \log\left(\frac{\text{Total Harmonic Distortion Power}}{\text{Signal Power}}\right)$$

$$= 10 \cdot \log\left(\sum_{k=2}^{\infty} X_k^2 / X_1^2\right) \qquad (1\text{-}56)$$

where $X_1$ is the rms value of fundamental and $X_k$ the rms value of the $k$-th harmonic component. Since there is an infinite number of harmonics the THD is usually calculated using the first 10-20 harmonics or until the harmonics can not be distinguished from the noise floor. The THD is sometimes defined as

$$THD = 10 \cdot \log\left(\frac{\text{Signal Power}}{\text{Total Harmonic Distortion Power}}\right) \qquad (1\text{-}57)$$

# *Methods*

1) Fourier transform of the output points – this is our project effort.
2) Evaluate with Numerical Polynomial of the data point
3) Evaluate the INL ( and DNL) – make sensible decision.

# *Results*

1) Most accurate
2) Accurate but tedious ( need to look at the errors
3) Very quick feeling on what's going on ( wors case only)

Numerical Polynomial

$y = f(x) \qquad P_n(x) = f(x)$

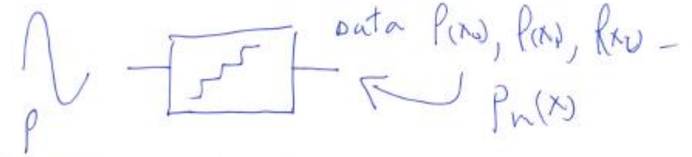
$P_n(x) = \sum L_n(x) f(x)$ where

$L(x) = \prod_{\substack{L \neq k \\ L=0}}^{n} \frac{x - x_i}{x_k - x_i}$ $\qquad$ error $= \frac{f^{n+1}}{(n+1)!}\prod(x - x_i)$

Lagrande polynomial

$$f(x) = f(x_0) + f(x_0, x_1)(x - x_0) + f(x_0, x_1, x_2)(x - x_0)(x - x_1) \ldots$$

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$ NewTon Form




build a polyn. from data.

construct $f(x) = 1 + \alpha_0 x + \alpha_1 x^2 + \alpha_2 x^3 + \ldots$

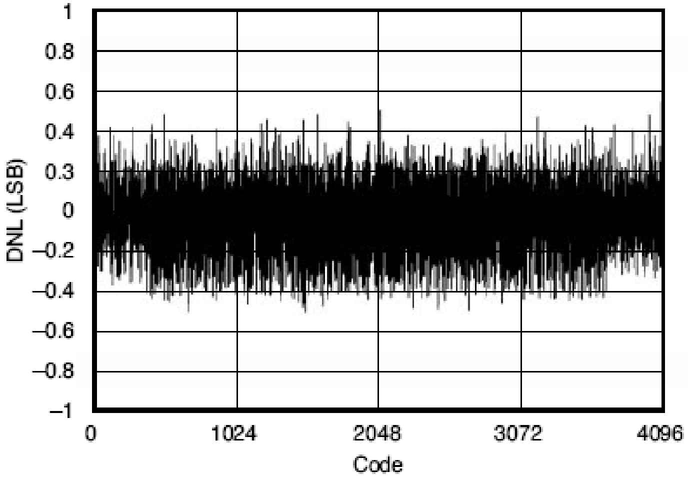
$x = \cos \omega t$

**Generate the outputs for each code.**
**You construct a polynomial using the numerical data you look at the coefitions of the polynomial with x=cos(wt).**

**DNL Definition**

**Differences between two adjacent output digital or analog compared to a step size of LSB weight.**

*Mathematically DEFENITION OF DNL*




$$DNL_i \stackrel{\Delta}{=} \frac{V_{i+1} - V_i}{VLSB} - 1 = INL_{i+1} - INL_i$$

DISTORTION: MISSING CODES, (INL/ DNL)

## INL Definition

**The Deviation of output code or output signal from straight line drawn from 0 and full scale**

after gain and offset are corrected is called Integral Non Linearity (INL)
INL leads to Harmonic distortions !

**Monotonic**: The output never decreases with increase of code or signal if INL<1 LSB the converter is monotonic- no missing codes.

*Mathematically DEFENITION OF INL*

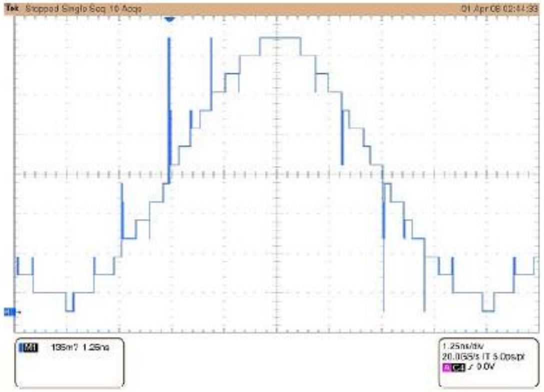
$$INL_i \stackrel{\Delta}{=} \frac{V_i - Voff}{VLSB} - i + \frac{1}{2}$$

# DNL/INL

**INL is measure of worst case distortion**
**However,**
**we do not know how and were the DNL/INL**
**is corrupted therefore only FFT**
**is accurate.**

**INL is a close indication of linearity (THD)**
**(remember should we extent the INL/DNL to AC )?**

<1 LSB INL implies less than 1 LSB DNL
<1 LSB DNL does not  implies less than 1 LSB INL



# INL related to DNL- YIELD

***THE RELATIONSHIP BETWEEN THE 2 :***

- Means that once we computed DNL, we can easily find INL using a cumulative sum operation on the DNL vector

$$INL_i = \sum_{k=-Nout_{Max}}^{i-1} DNL_k$$

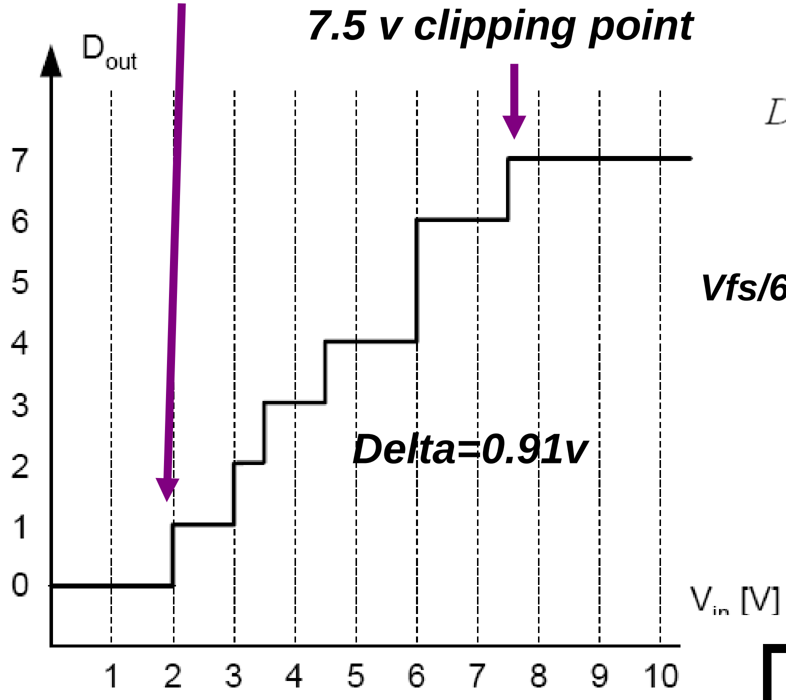
$$DNL_i \stackrel{\Delta}{=} \frac{V_{i+1} - V_i}{VLSB} - 1 = INL_{i+1} - INL_i$$

**If INL/DNL are due to elements in the analog blocks not linear/equal they are either systematic we made mistake in the design or mismatch in silicon ( resistors/current source) -→ YIELD IS EFFECTED – calculate it**

# INL/DNL- in class example

***2v min point point***

***7.5 v clipping point***




$$DNL_i \stackrel{\Delta}{=} \frac{V_{i+1} - V_i}{VLSB} - 1$$

***Vfs/6steps= (7.5-2)/6 =0.91***

***1/0.91-1=0.09***

***0.5/0.91-1=-0.45***

***1.5/0.91-1=-0.45***

***V(i+1) – V(i)***

| | |
|---|---|
| 0 | undefined |
| 1 | 1 |
| 2 | 0.5 |
| 3 | 1 |
| 4 | 1.5 |
| 5 | 0 |
| 6 | 1.5 |
| 7 | undefined |

***We have 6 steps and 7.5 v clipping point***

***0.09-0.45=-0.36***

***-0.36+0.09=-0.27***

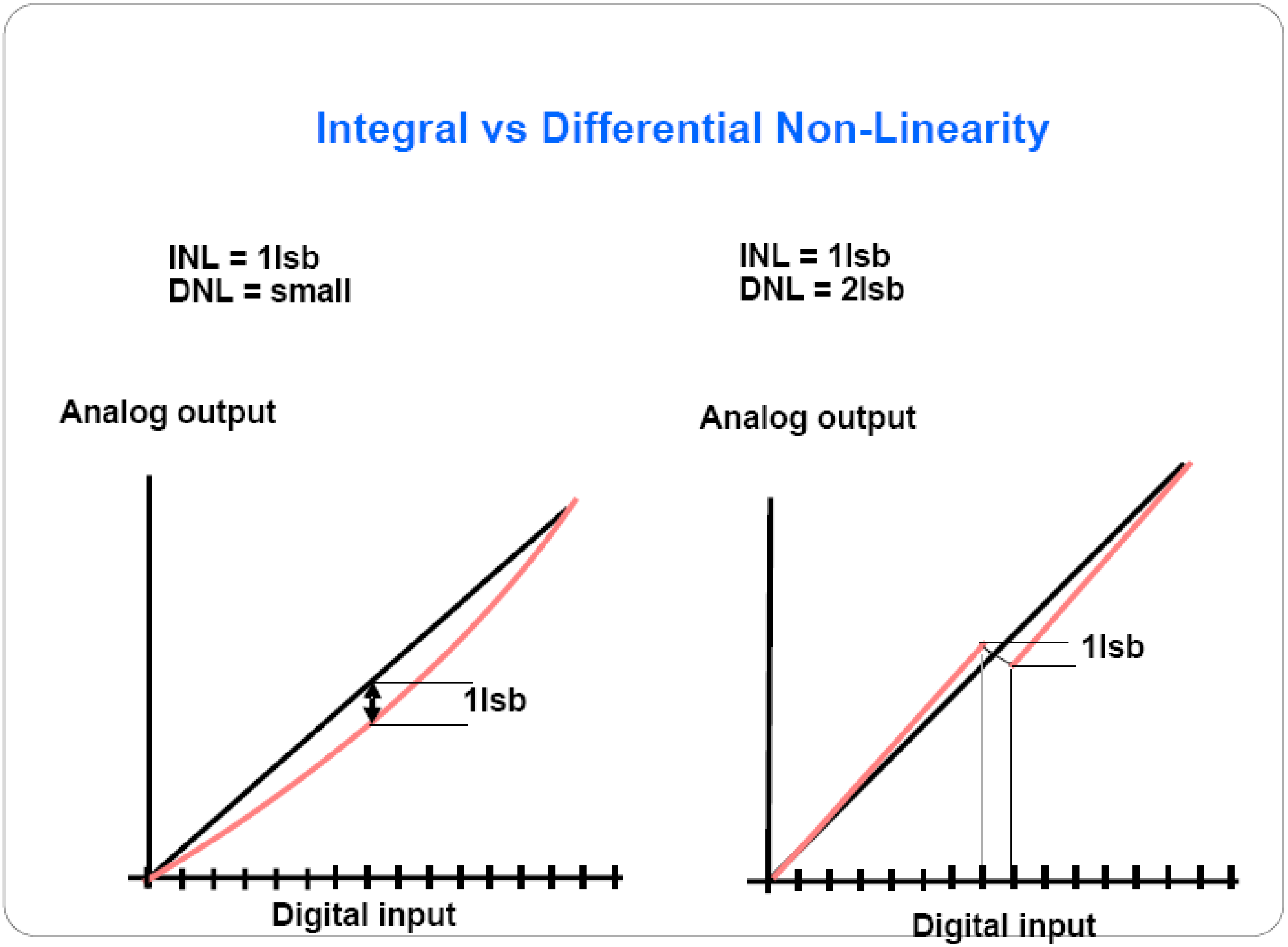
***-0.27+0.64=0.37***

***0.37-1=-0.64***

***example : (Source: B.Murmann Stanford)***

| Code (k) | DNL [LSB] | INL (LSB |
|---|---|---|
| 1 | 0.09 | 0 |
| 2 | -0.45 | 0.09 |
| 3 | 0.09 | -0.36 |
| 4 | 0.64 | -0.27 |
| 5 | -1.00 | 0.36 |
| 6 | 0.64 | -0.64 |
| 7 | undefined | 0 |

***0.37***




**<1 LSB DNL does not implies less than 1 LSB INL**

# Summary

*In general our object is to keep all mismatches to below +/-1/2LSB*

# ENOBS

## SNDR is the measured value

**SNDR is measure of effective resolution ( "real" of the converter**
**N- Quantization**
**D- Harmonics**

**DFINITION OF ENOBS**

**Linearity test:**
**1.With a Line set by end points (on occasion is best fit)- DC measure – can we extend to AC ?**

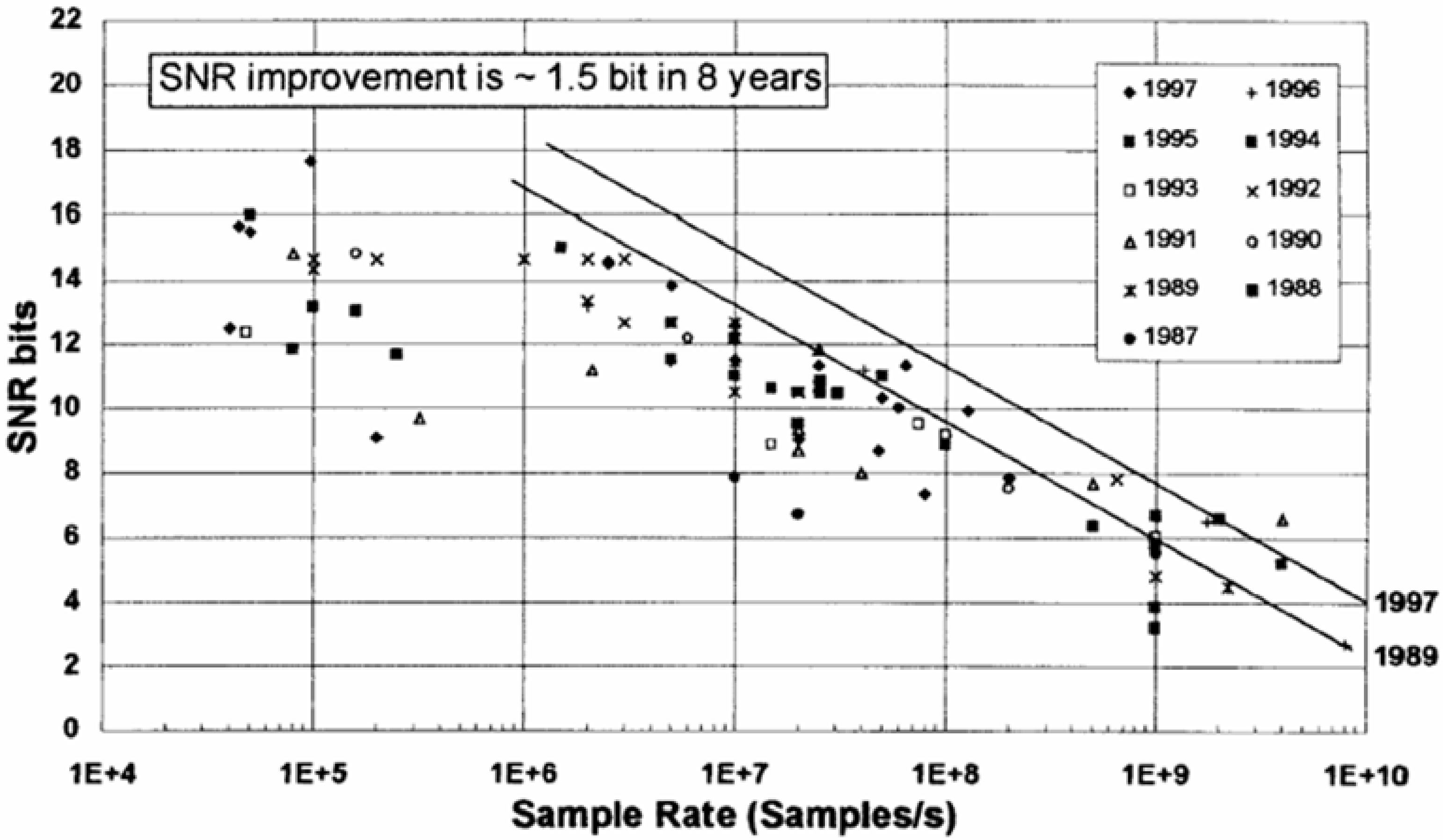
**1.FFT the output – will tell it all.**
**ENOB is the effective number of bits**

**++thermal noises..**

$$ENOBS(bit) \equiv SNDR(effective) - 1.76 / 6.02)$$

$$ENOB = \frac{S}{N + N_{INL}} = \frac{S}{N} \cdot \frac{N}{N + N_{INL}}$$

# ENOBS improvmrnts..




R.H. Walden, "Analog-to-digital converter survey and analysis," IEEE Journal on Selected Areas in Communications, vol. 17, no. 4, pp. 539-550, April 1999.

***1.5bit/8yrs – slow improvement..***

## Definition of SFDR

***Spurious Free Dynamic Range of a converter.***

*Is the ratio of the largest Harmonic component to the signal compo*

*It's a good measure for differential structures and to evaluate mism*
*CAN BE DONE AC TO BE EVEN CLOSER TO REALITY ( MAX B*

gated via a Taylor expansion of the $i_o = f(v_i)$ function in the equilibrium point:

$$i_o(t) = \alpha_1 v_i(t) + \alpha_3 v_i^3(t) + \alpha_5 v_i^5(t) + \alpha_7 v_i^7(t) + \ldots \qquad (1)$$

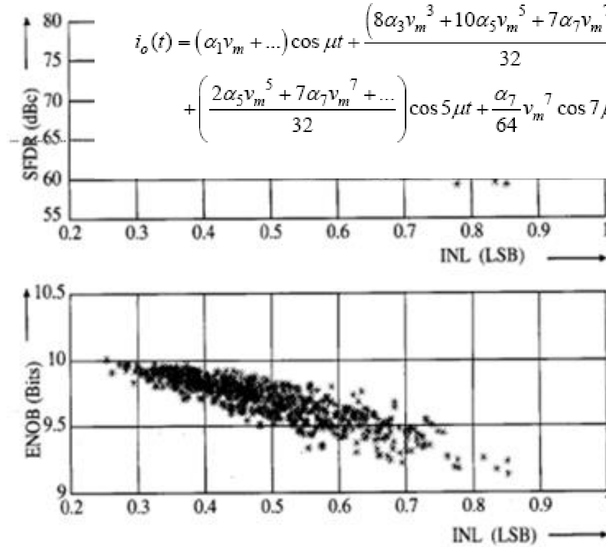
where the $\alpha_i$ parameters are determined from the particular circuit implementation. For a harmonic input of the type: $v_i(t) = v_m \cos \mu t$, and after grouping of the frequency components, (1) can be rewritten in the form:

$$i_o(t) = (\alpha_1 v_m + \ldots)\cos \mu t + \frac{(8\alpha_3 v_m^3 + 10\alpha_5 v_m^5 + 7\alpha_7 v_m^7 + \ldots)}{32}\cos 3\mu t + \left(\frac{2\alpha_5 v_m^5 + 7\alpha_7 v_m^7 + \ldots}{32}\right)\cos 5\mu t + \frac{\alpha_7}{64} v_m^7 \cos 7\mu t + \ldots \qquad (2)$$

$$i_o(t) = \alpha_1 v_i(t) + \alpha_3 v_i^3(t) + \alpha_5 v_i^5(t) + \alpha_7 v_i^7(t) + \ldots \qquad (1)$$

where the $\alpha_i$ parameters are determined from the particular circuit implementation. For a harmonic input of the type: $v_i(t) = v_m \cos \mu t$, and after grouping of the frequency components, (1) can be rewritten in the form:

$$i_o(t) = (\alpha_1 v_m + \ldots)\cos \mu t + \frac{(8\alpha_3 v_m^3 + 10\alpha_5 v_m^5 + 7\alpha_7 v_m^7 + \ldots)}{32}\cos 3\mu t + \left(\frac{2\alpha_5 v_m^5 + 7\alpha_7 v_m^7 + \ldots}{32}\right)\cos 5\mu t + \frac{\alpha_7}{64} v_m^7 \cos 7\mu t + \ldots$$




***Source: R.V. Plassche***

$$SFDR(dB) = -20\log(|INL|\,2^{-N_{bits}} + 2^{-1.5N_{bits}}).$$

***Remember: The 1.5 comes from the "perfect" converter.***

*In general we will try to keep all mismatches to below +/-1/2LSB*

# *FOM*

**HOW TO DEFINE A GOOD ADC?**
**Figure of Merit (F.O.M)**
**It combines “all“ parameters in one. !**

# FOM

Energy per conversion step! (Pico joules/conversion)

**Definition 1**

**How to measure how good is a converter**

**Or the inverse (usually for DACs)**

**Definition 2.**

$$FoM = \frac{P}{2^{ENOB} \times 2 \times ERBW}$$

$$\text{Energy/Decision} = \frac{Power}{SamplingRate \cdot 2^{Nbit}}$$

Energy per conversion step! (Pico joules/conversion)

P = Power (does Added element included PLL?)

ENOB = Effective number of bits but at full BW or DC?

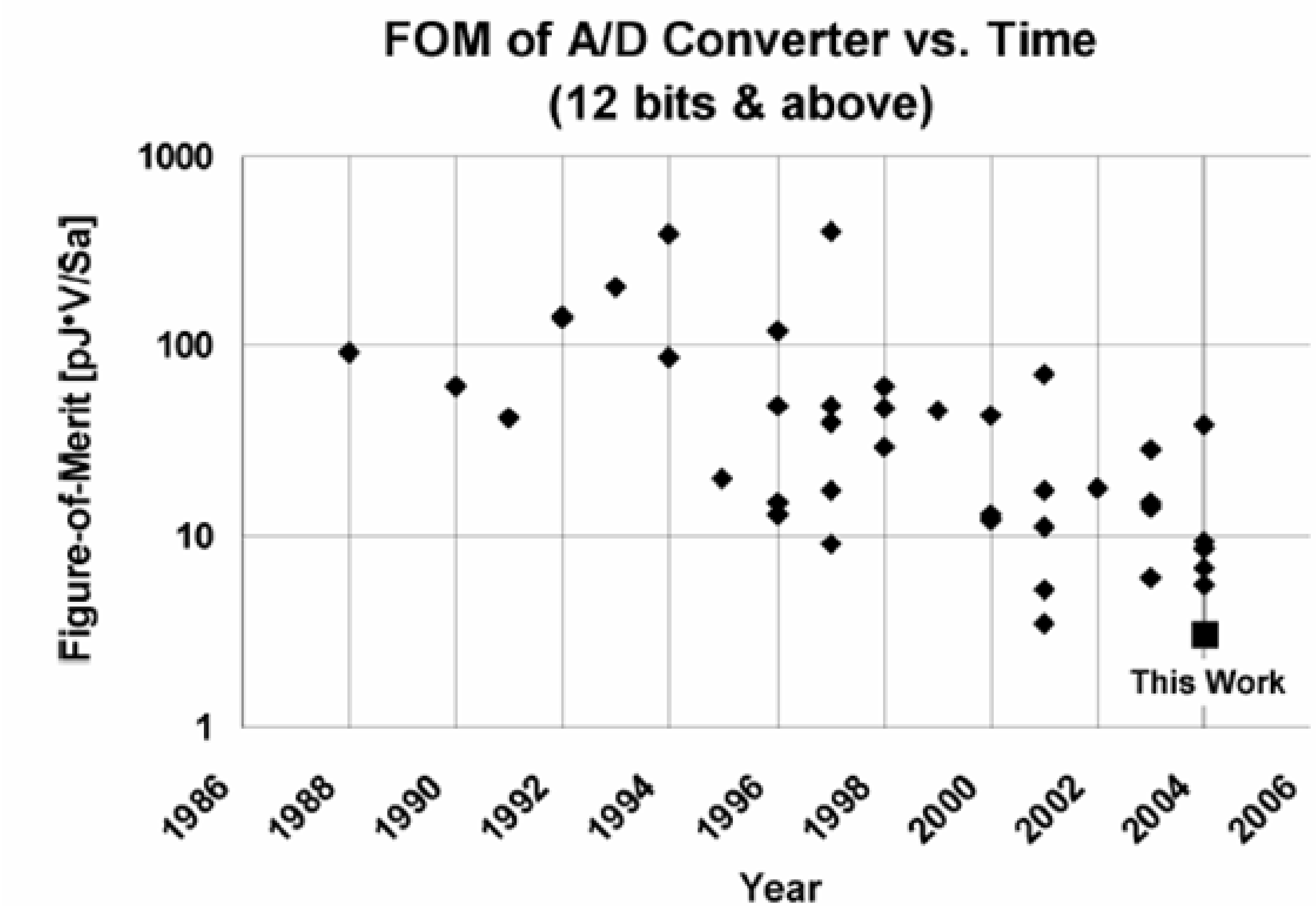
No Area? (Sometime you multiply by Vcc)

Grain of salt: Because of technology and specs are different factor

**Number below 1 are good**! (..12b/40Mw/5MHz)…

| | All designs | | High Frequency ((above 500 MHz | |
|---|---|---|---|---|
| | Averag e | Media n | Averag e | Median |
| Energy per [decision [pJ | 1.65 | 0.84 | 1.71 | 1.73 |
| Figure of [Merit [pJ*V | 7.40 | 5.48 | 5.55 | 5.58 |

# *FOM example.*




Xun Chiu; Gray, P.R.; Nikolic, B.,"A 14-b 12-MS/s CMOS pipeline ADC with over 100-dB SFDR", IEEE Journal of Solid-State Circuits, , Volume: 39 , Issue: 12 , Dec. 2004

# **Key:** Linearity (INL) reduction on SNRD(ENOBs)

### *ENOB SFDR Vs. INL model*

***In reality since the converter is not accurate the INL/DNL can be inside the +/- 0.5 lsb but the converter is not n bit converter !***

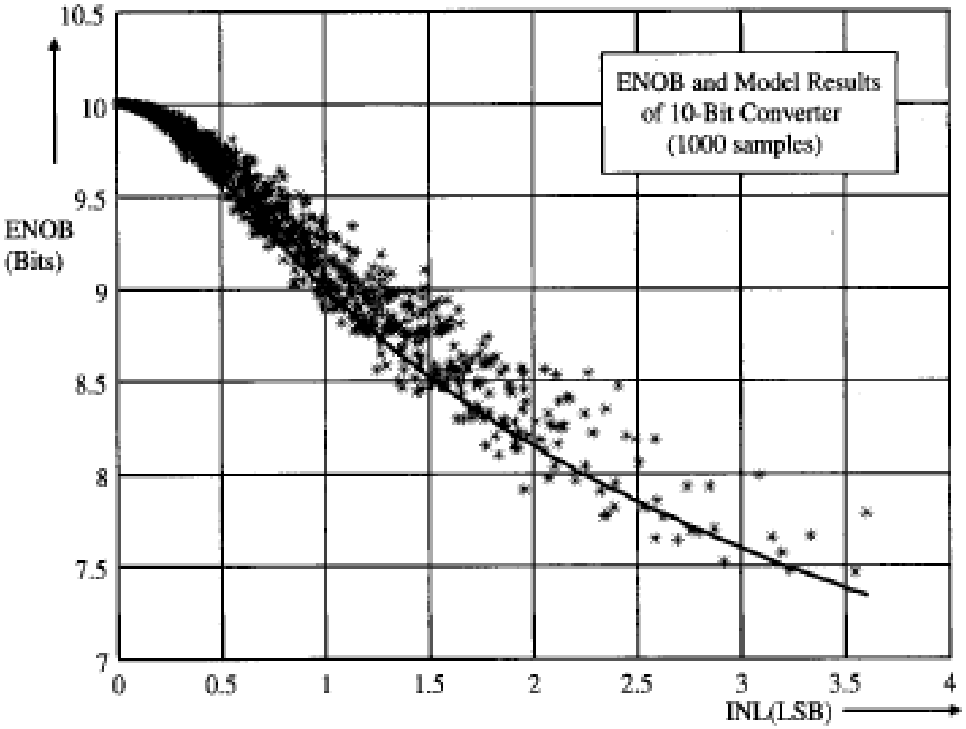



Figure 2.19: INL versus ENOB reduction model

$$V_{out} = V_{in}(1 + INL(LSB)).$$

$$n_{reduction} = \frac{\log\left(1 + 3 * |INL|^2\right)}{2\log 2}.$$

***Source: R.V. Plassche***
*Page 76.*

***1000 different converters***
*All "10 bits"*

***In reality INL of LSB does not means the converter in n bit but more like ~ n-1.***

# Summary

**error**

**Possible contributor**

QUANTIZATION NOISE → Design decision – how many bits

Distortions: DNL, INL, missing codes (THD) → Mismatches in the design including gnd; Maybe power supply or substrate noises

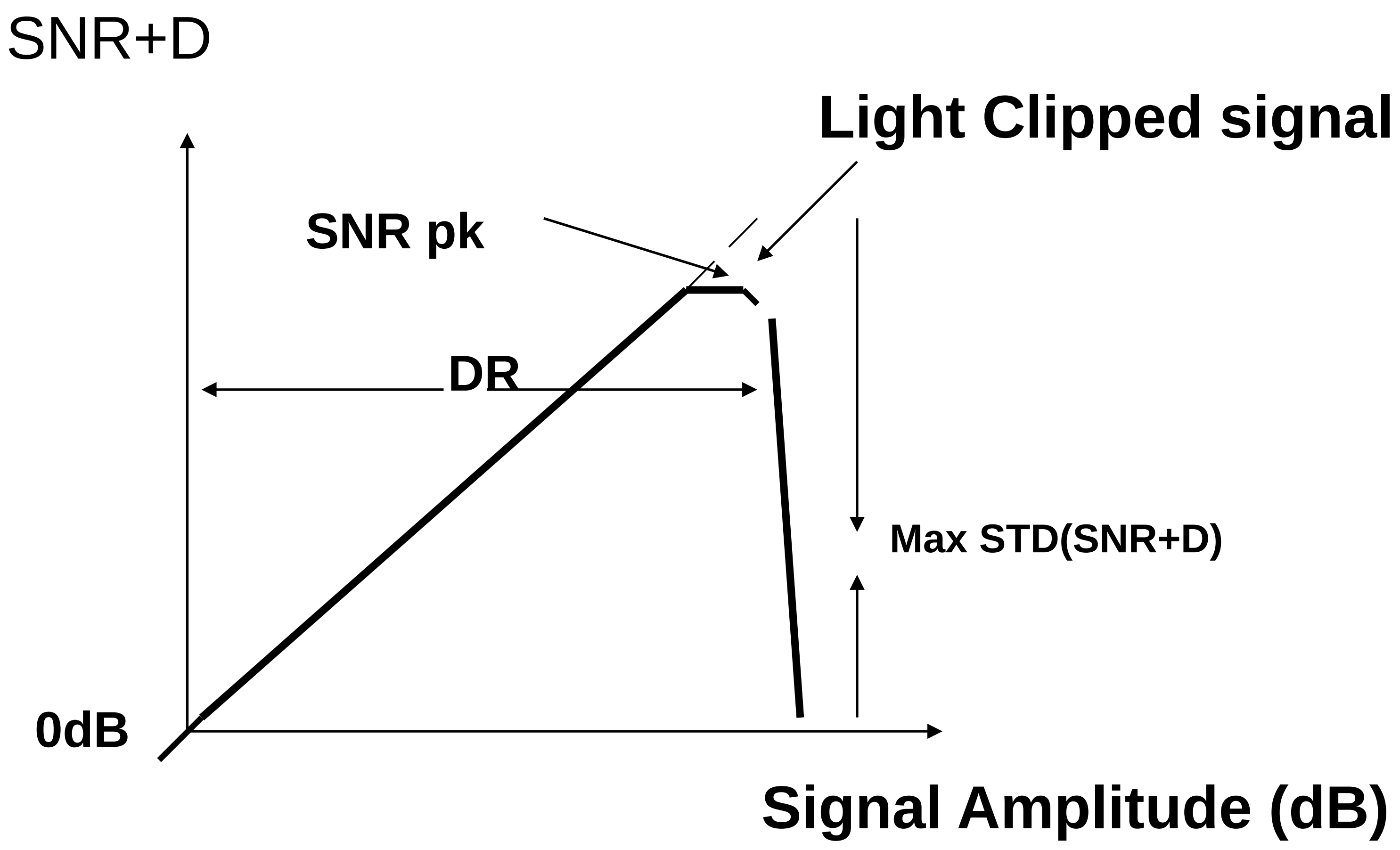
SNR → Thermal noises, 1/f noises, clock jitter (and quantization error); Design decision – how many bits

SNRD

FOM → **Power Issue: Chosen architecture to meet all the above, and optimum design**

# Misc, Added Notes

DR definition = maximum signal/min signal( were its berried in noise) in power.




**DR may be bigger than SNR Pk**
**DR ≠ SNRpk**

## Sampling- A Step Back At Fourier Transform

Fourier Series -

IF $x(t)$ = periodic, period $f_0 = \frac{1}{T_0}$

$$C_x(nf_0) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-j2\pi n f_0 t}\, dt$$

---

$x(t)$ Can be written as.

$$\sum_{n=-\infty}^{\infty} C_x(nf_0)\, e^{+j2\pi n f_0 t} \qquad \infty < t < \infty$$

$$= A_0 + \sum_{n=1}^{\infty} A_n \cos 2\pi n f_0 t + \sum_{n=1}^{\infty} B_n \sin 2\pi n f_0 t$$

$$C_x(nf_0) = A_n - jB_n.$$

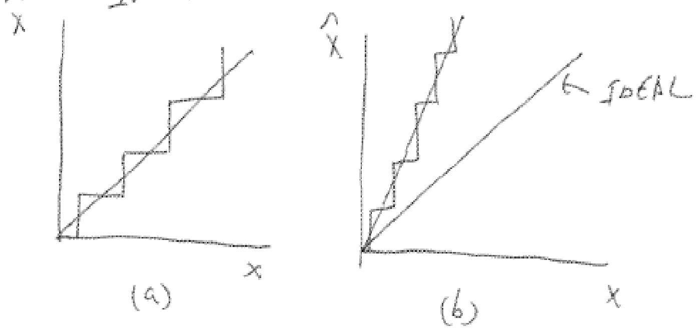
Any periodic signal can be constructed from sum of sine waves.

The power (or P.S.D) - Density

is $S_x = \int_{-\infty}^{\infty} \underbrace{|C_x(nf_0)|^2\, \delta(f - nf_0)}_{G_x(f)}\, df = \underline{\underline{Power}}$

also $P = \frac{1}{T}\int_{-T/2}^{T/2} |x^2(t)|^2\, dt$

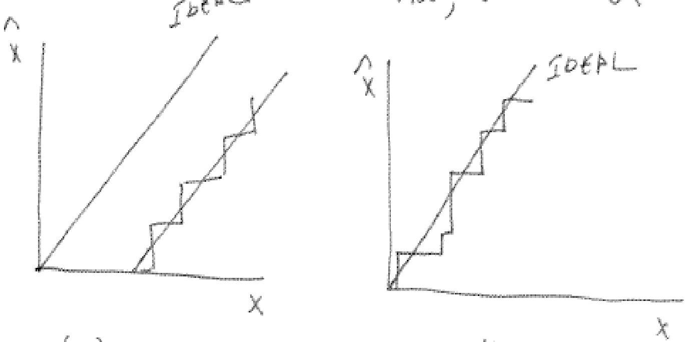
IDEAL ERRORS IN CONVERTERS




(a) PERFECT

(b) GAIN ERROR

IDEAL TRIM VREF, OR code · α

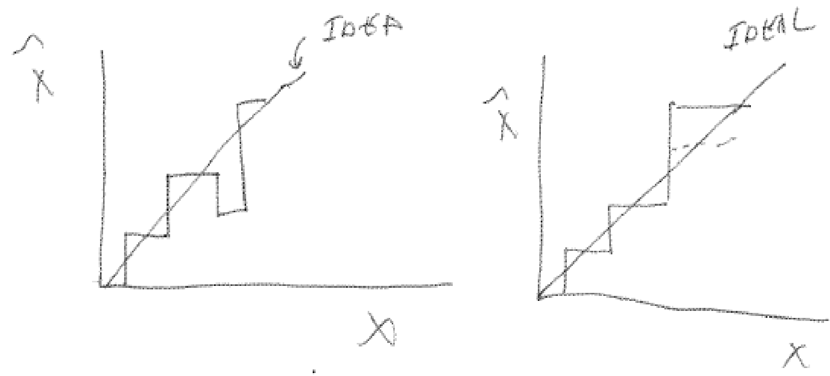



(c) offset errors

(d) Non Linearity

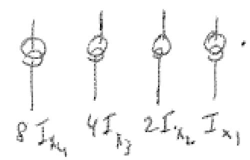
FOR ADC X produces code $\hat{X}$
FOR DAC $\hat{X}$ produces code X

EXTREME LINEARITY ERRORS




non MONOTONICITY

missing code

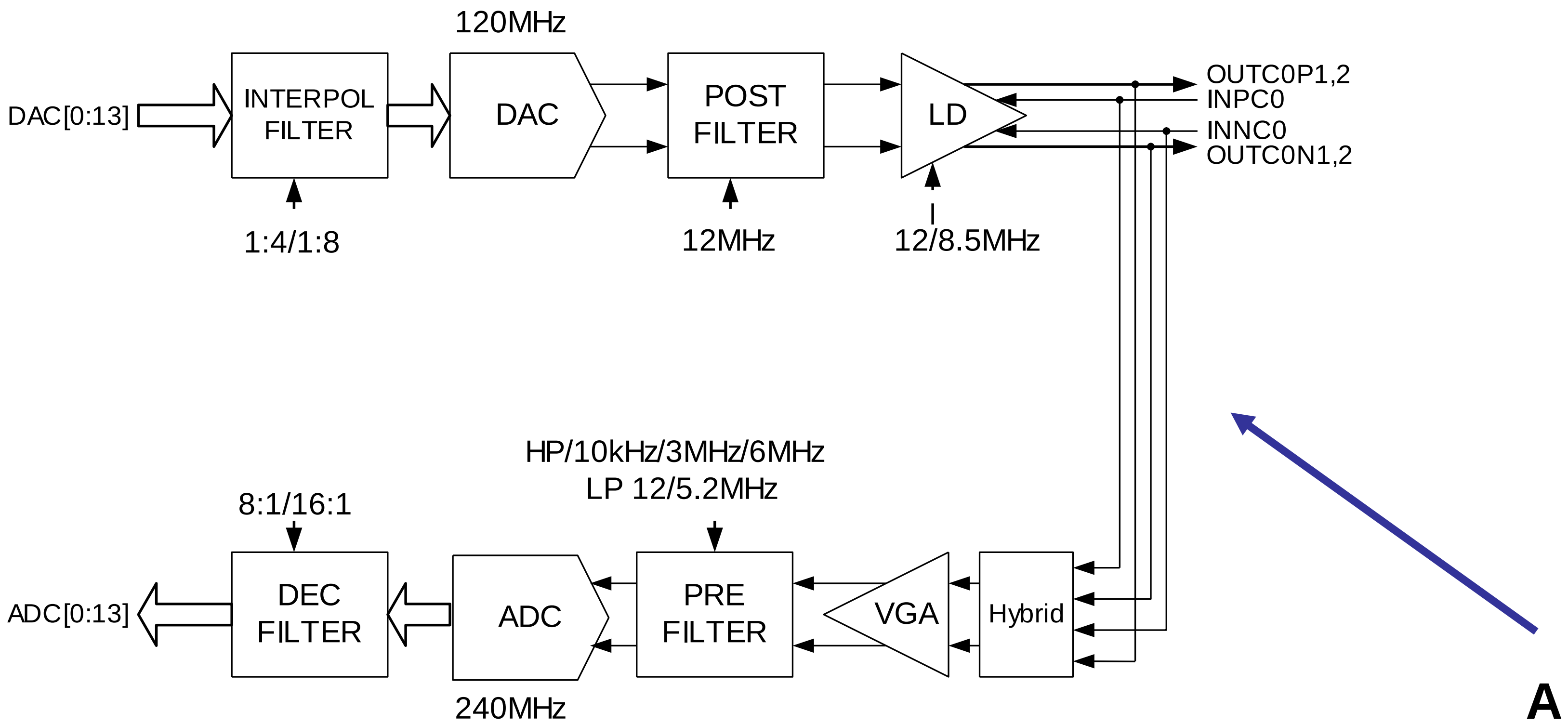


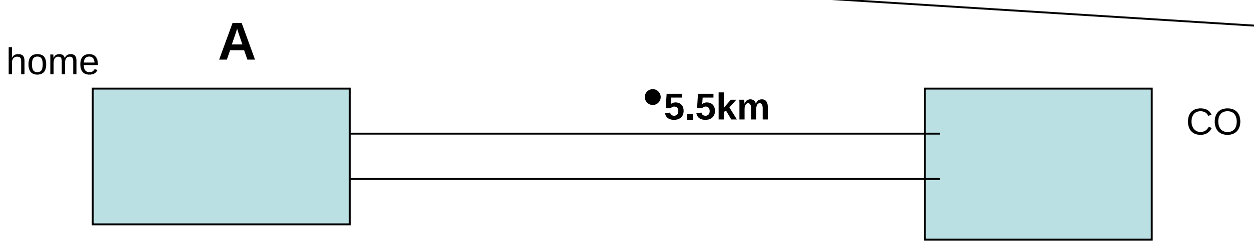

0 1 1 1 = 7I
1 0 0 0 8I

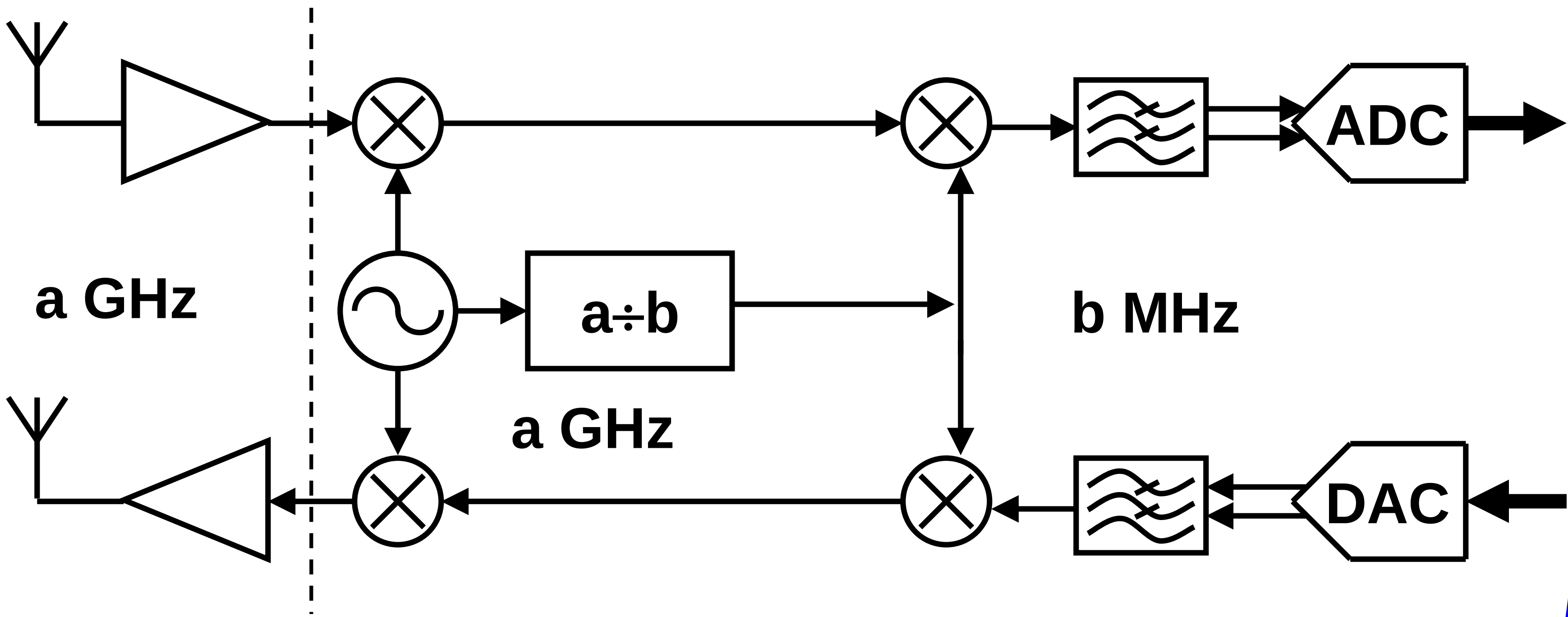
8I ARE NOT MADE THE SAME AS $7I_{x_{1-3}}$!
and could end up less than $7I_{x_4}$

Missing Codes.

# Example: xDSL AFE Architecture






- Antenna length forces high frequency mod.

**Old codecs, voice music..**
**DSL front ends – multi bit , one bit(CDRs)**
**Wirless ADCs**
**Sensing : X ray detection ultrasounds..**

**DSP**

# End lecture 2 (and part of 3)